STRUT-AND-TIE MODELING PROVISIONS

WHAT, WHEN, AND HOW?

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PURDUE
UNIVERSITY

WHAT IS STRUT-AND-TIE MODELING (STM)?

- **Lower-bound (i.e., conservative) design method for reinforced concrete structures**
  - **Design of *D-regions* ("D" = discontinuity or disturbed)**

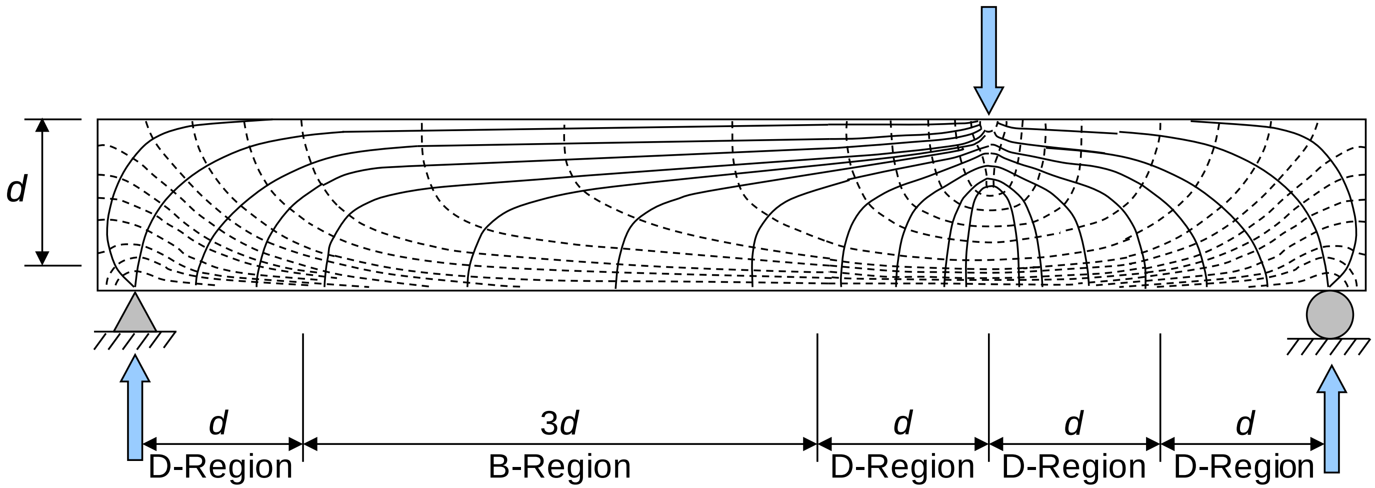


Figure: Stress trajectories within flexural member

- **D-regions vs. *B-regions* ("B" = beam or Bernoulli)**

D-REGIONS VS. B-REGIONS

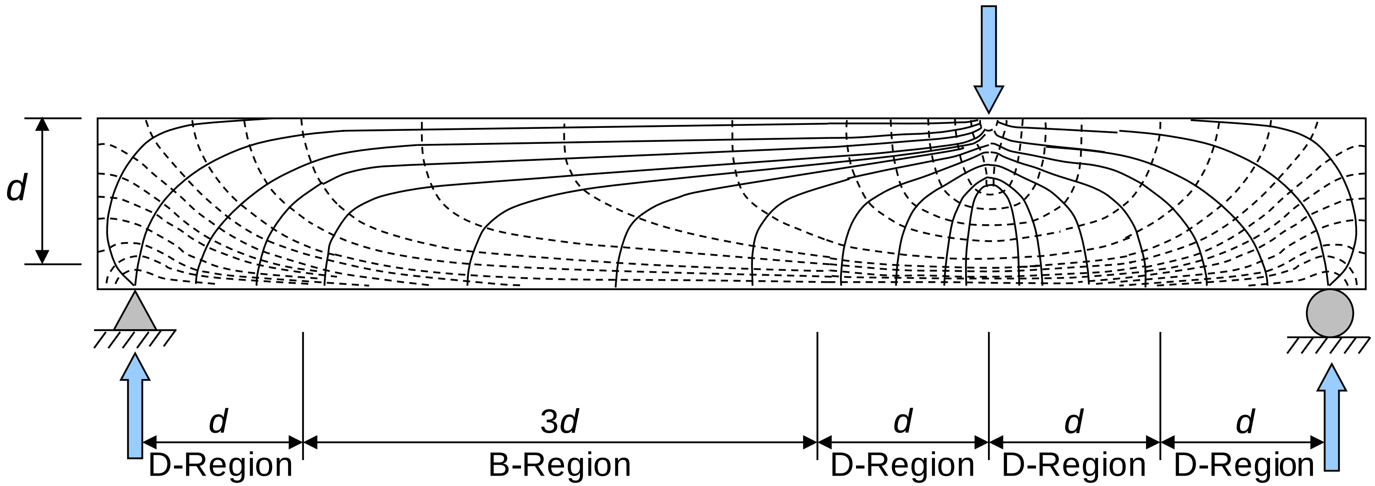
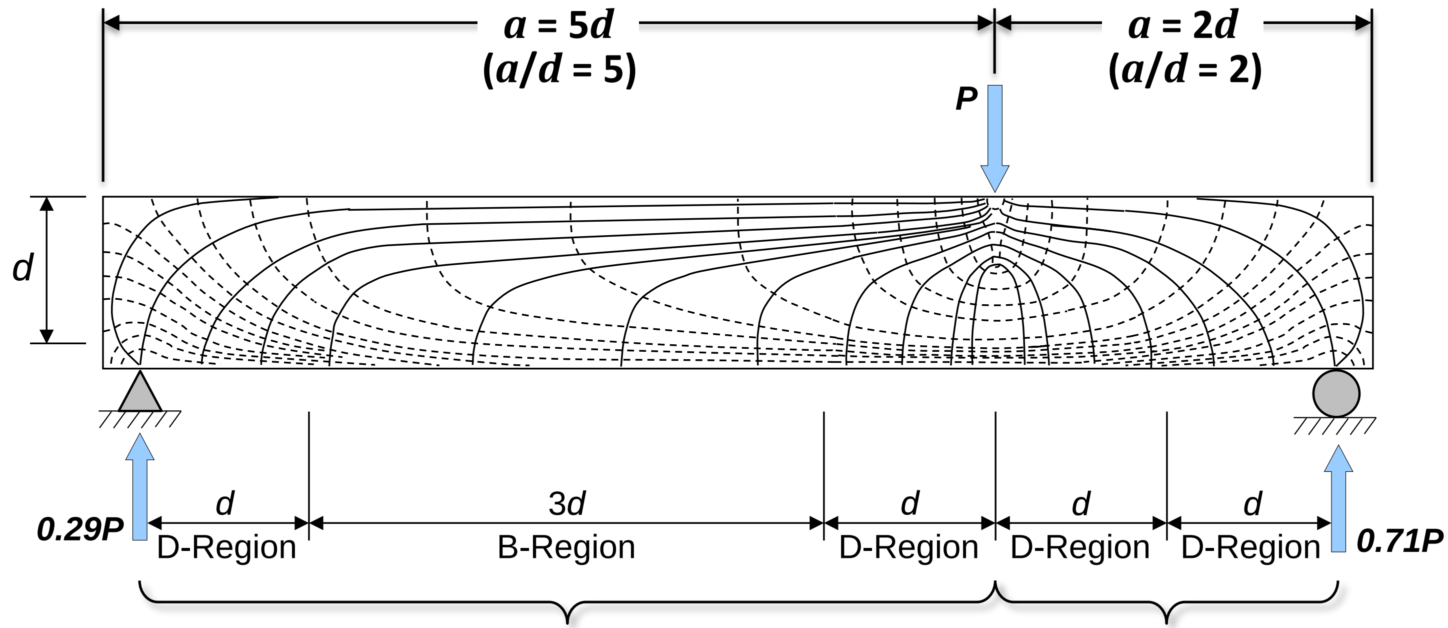


Figure: Stress trajectories within flexural member

- **D-regions**
  - **Within d of load or $\underbrace{\text{geometric discontinuity}}_{\text{Frame corner, dapped end, opening, corbel}}$ (St. Venant's Principle)**
  - **Nonlinear distribution of strains**
- **B-regions**
  - **Linear distribution of strains**
  - **Plane sections remain plane**

WHEN DO YOU NEED TO USE STM?



Dominated by
Sectional Behavior
($a/d \geq$ 2.0 to 2.5)

Sectional Design Procedure is Valid

Dominated by
Deep Beam Behavior
($a/d \leq$ 2.0 to 2.5)

Sectional Design Procedure is Invalid
∴ Use STM

Shear-span-to-depth ratio

EXISTING STRUCTURES: FIELD ISSUES

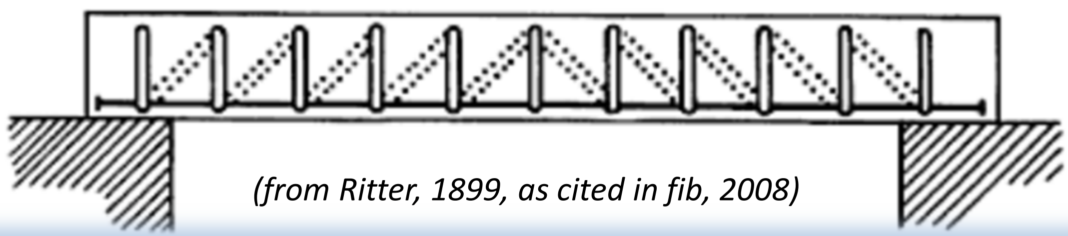
EXISTING STRUCTURES: FIELD ISSUES

Retrofit

EXISTING STRUCTURES: FIELD ISSUES

STRUT-AND-TIE MODELING PROVISIONS

Development of truss analogy for the behavior of reinforced concrete structures (Ritter, 1899; Mörsch, 1902)



(from Ritter, 1899, as cited in fib, 2008)

Development and refinement of STM among European researchers (Schlaich and others)

STRUT-AND-TIE MODELING PROVISIONS

STM introduced into AASHTO LRFD provisions in 1994

STM introduced into ACI 318 provisions in 2002

Routine implementation of STM provisions has been impeded due to uncertainty within the engineering community

STRUT-AND-TIE MODELING RESEARCH

Brown et al.
(2002-2006)

Design for Shear Using STM

Birrcher et al.
(2006-2009)

Strength and Serviceability Design of Deep Beams Using STM

Williams et al.
(2009-2012)

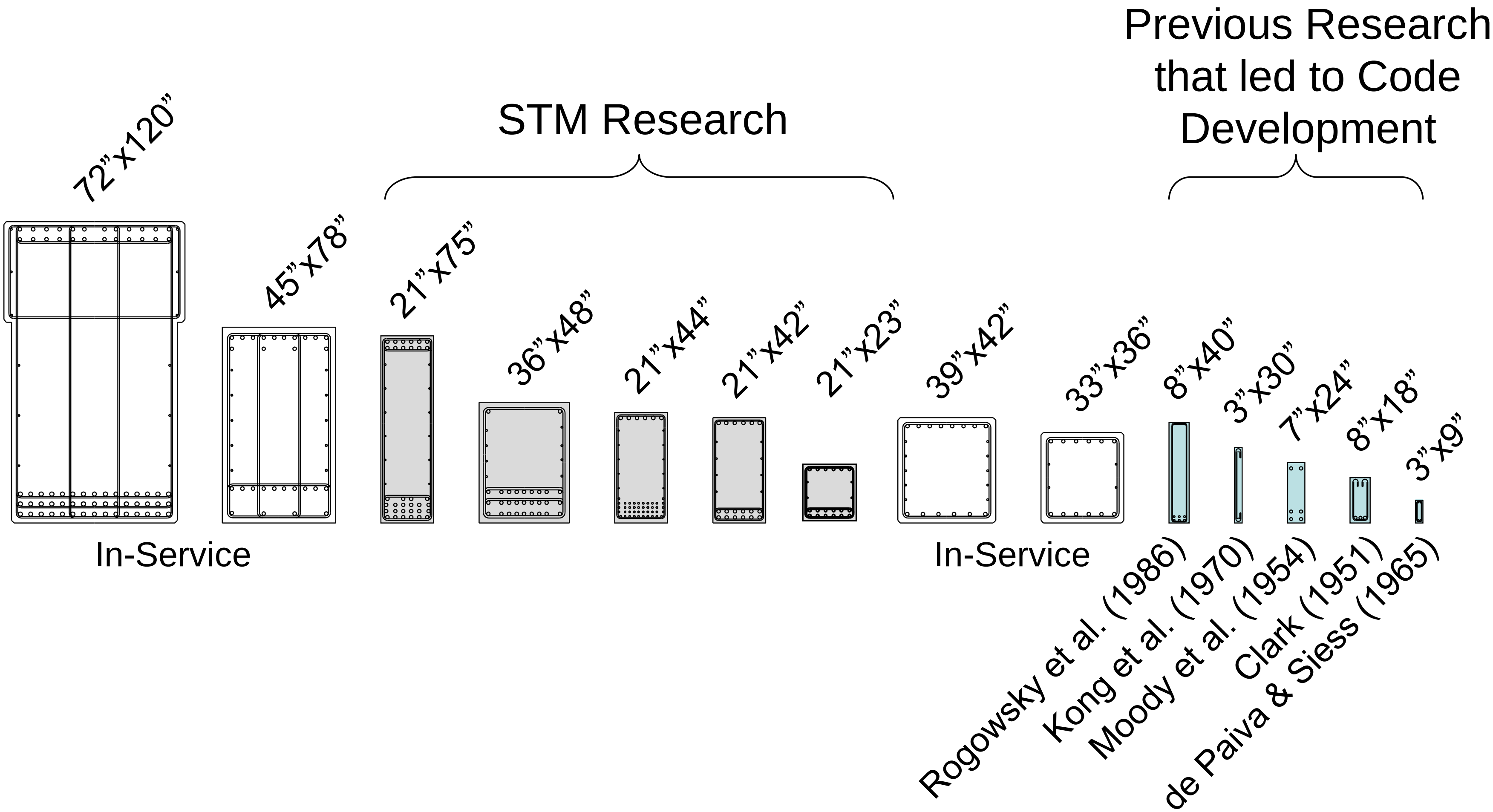
STM Guidebook with Design Examples

Larson et al.
(2009-2013)

Strength and Serviceability Design of Inverted-T Beams Using STM

DEEP BEAM EXPERIMENTAL WORK

DEEP BEAM EXPERIMENTAL WORK



INVERTED-T EXPERIMENTAL WORK

STRUT-AND-TIE MODELING PROVISIONS

STM introduced into AASHTO LRFD provisions in 1994

↓

STM introduced into ACI 318 provisions in 2002

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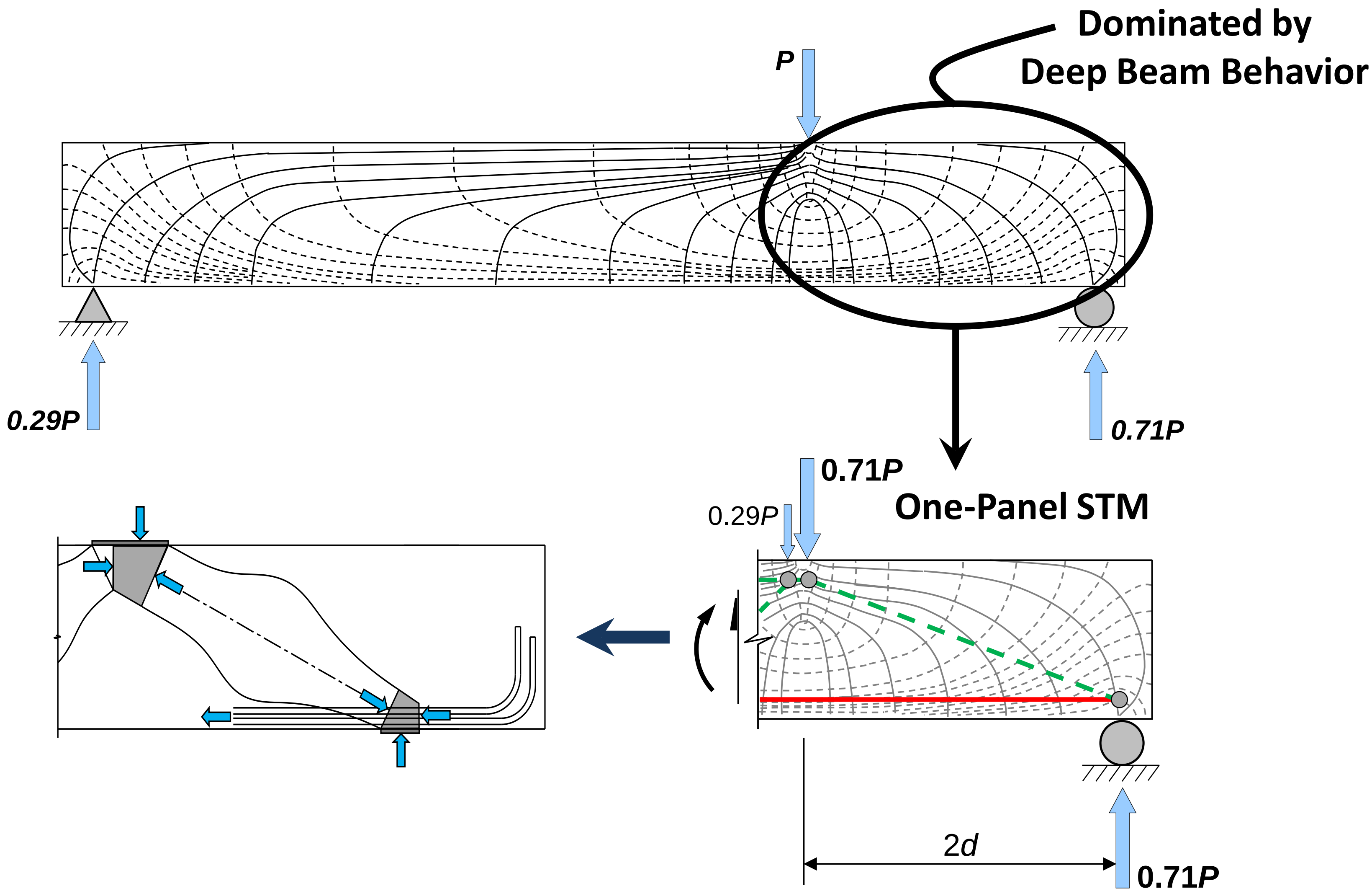
Re-write of STM provisions in AASHTO LRFD 2016 Interim Revisions

HOW DO YOU USE STM?

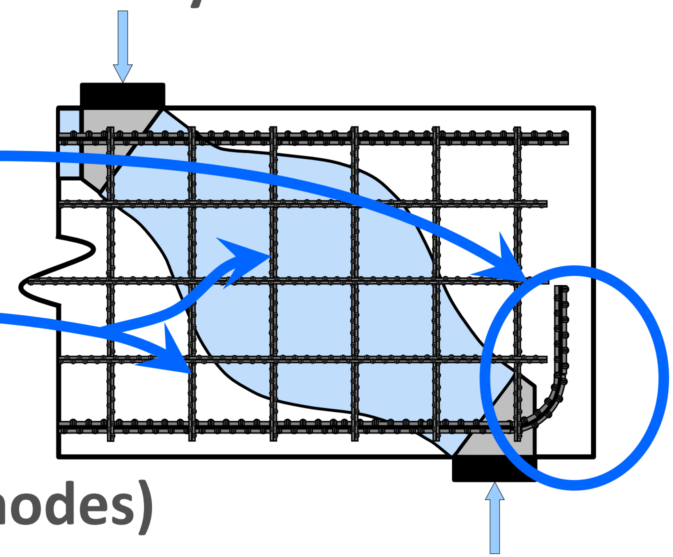


STM FUNDAMENTALS

STM is a lower-bound (i.e., conservative) design method, provided that:

1. Strut-and-tie model is in equilibrium with external forces (and internal equilibrium is satisfied)

2. Concrete element has sufficient deformation capacity to allow distribution of forces assumed by the STM
   - Key detailing requirements:
     - ✓ Proper anchorage of reinforcement
     - ✓ Distributed orthogonal reinforcement

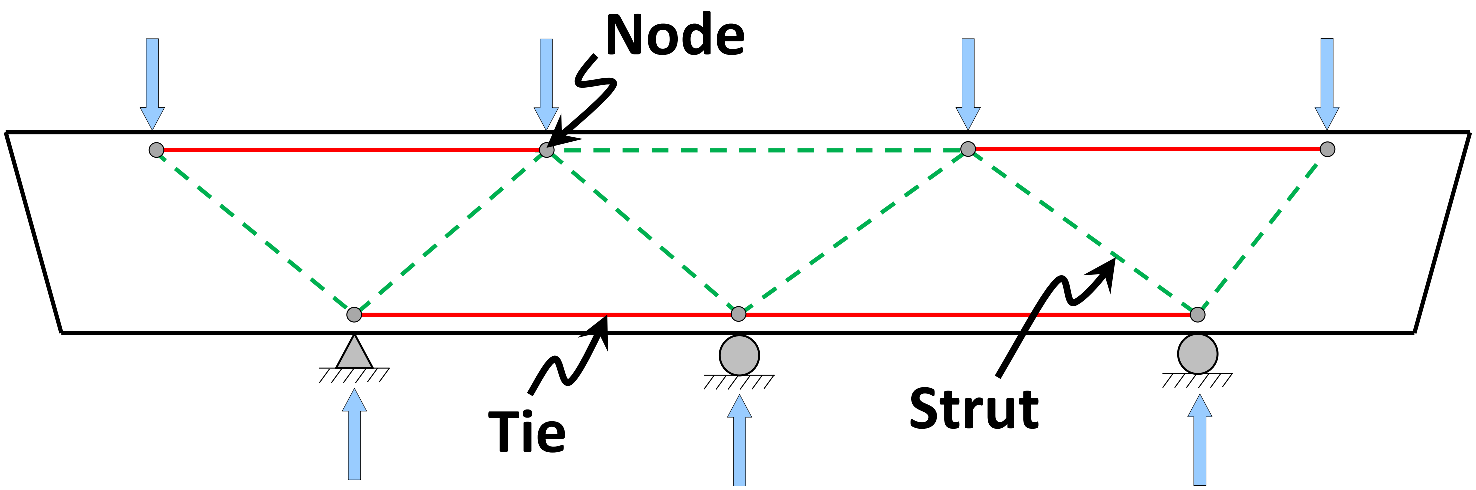


3. Strength is sufficient (ties and nodes)

STM FUNDAMENTALS

Three parts to every STM:

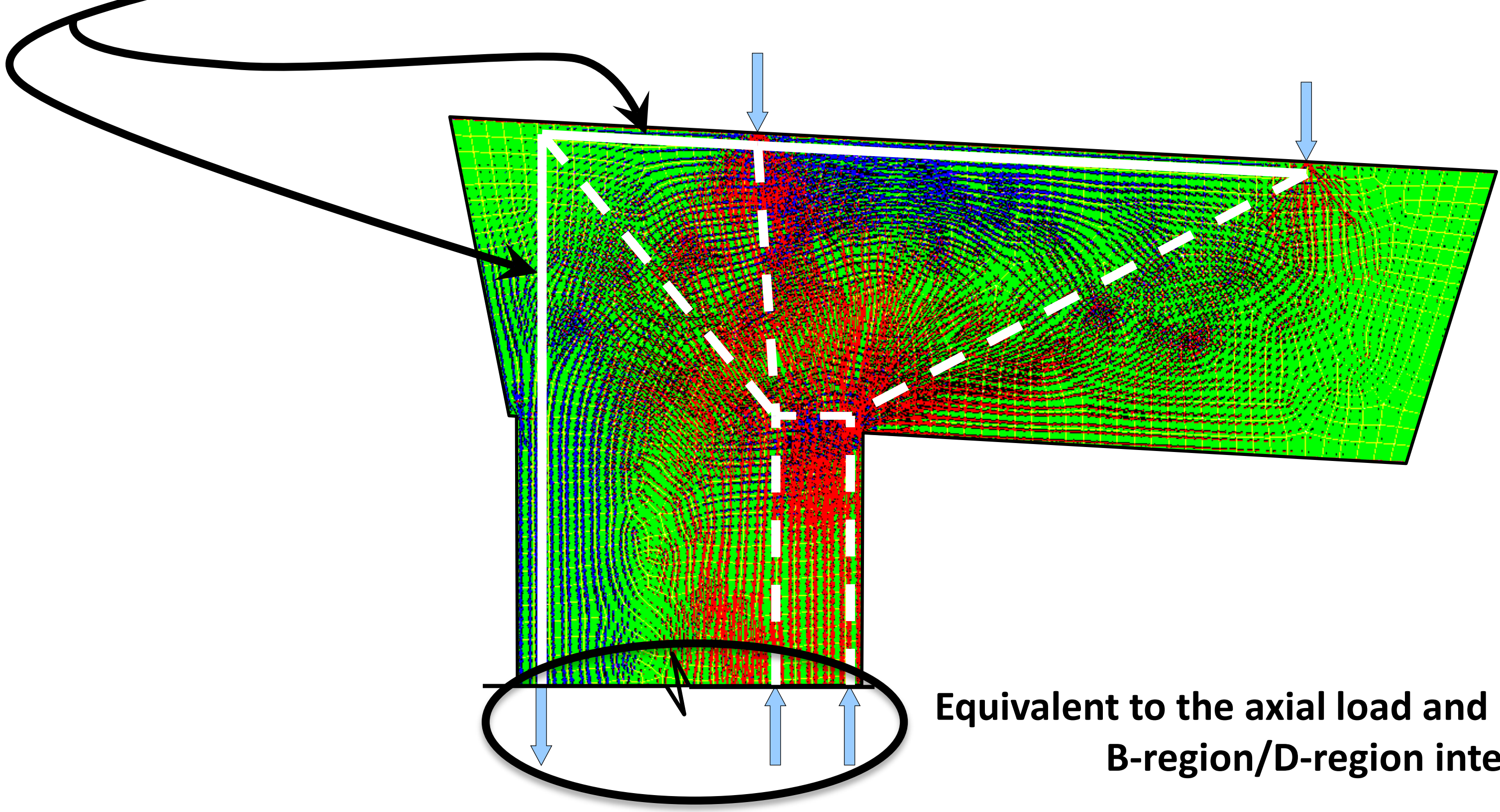
Struts **Ties** **Nodes**



STM FUNDAMENTALS

Place struts and ties according to “flow” of forces indicated by an elastic analysis

Ties must be located at the centroid of the reinforcing bars

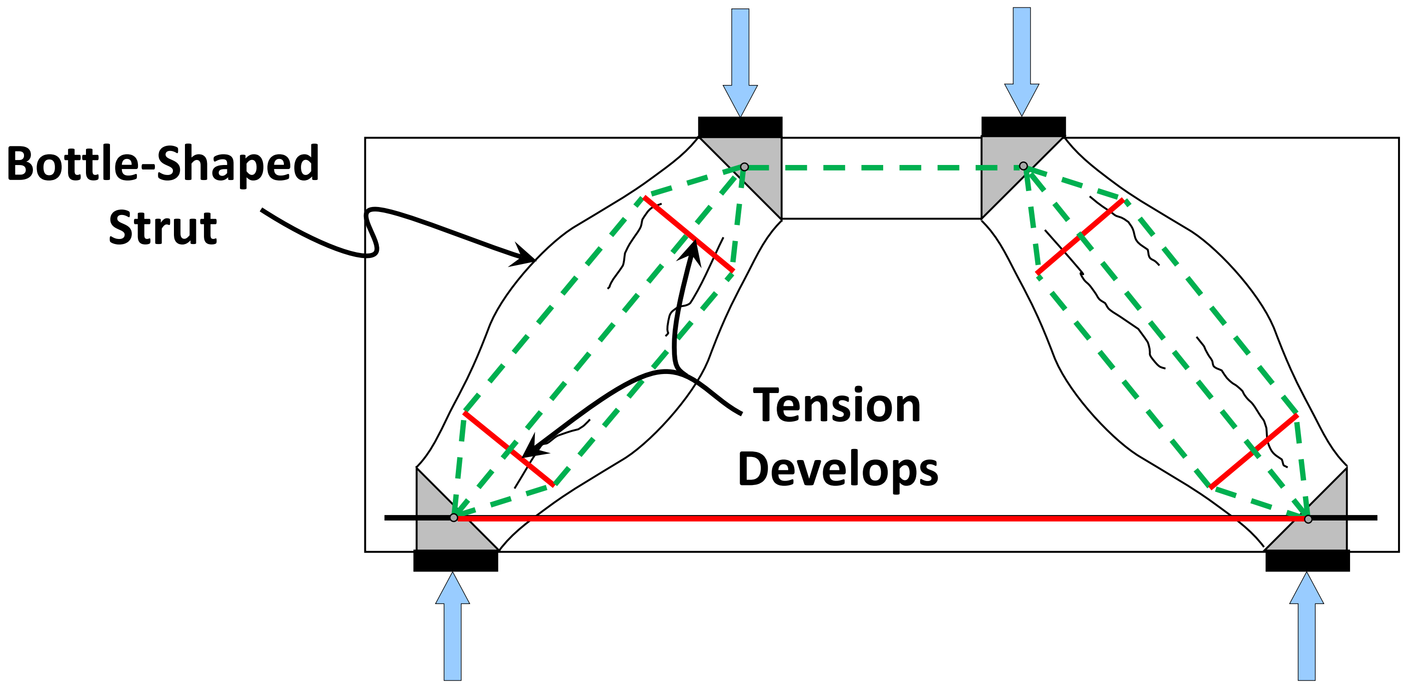


Equivalent to the axial load and moment at the B-region/D-region interface

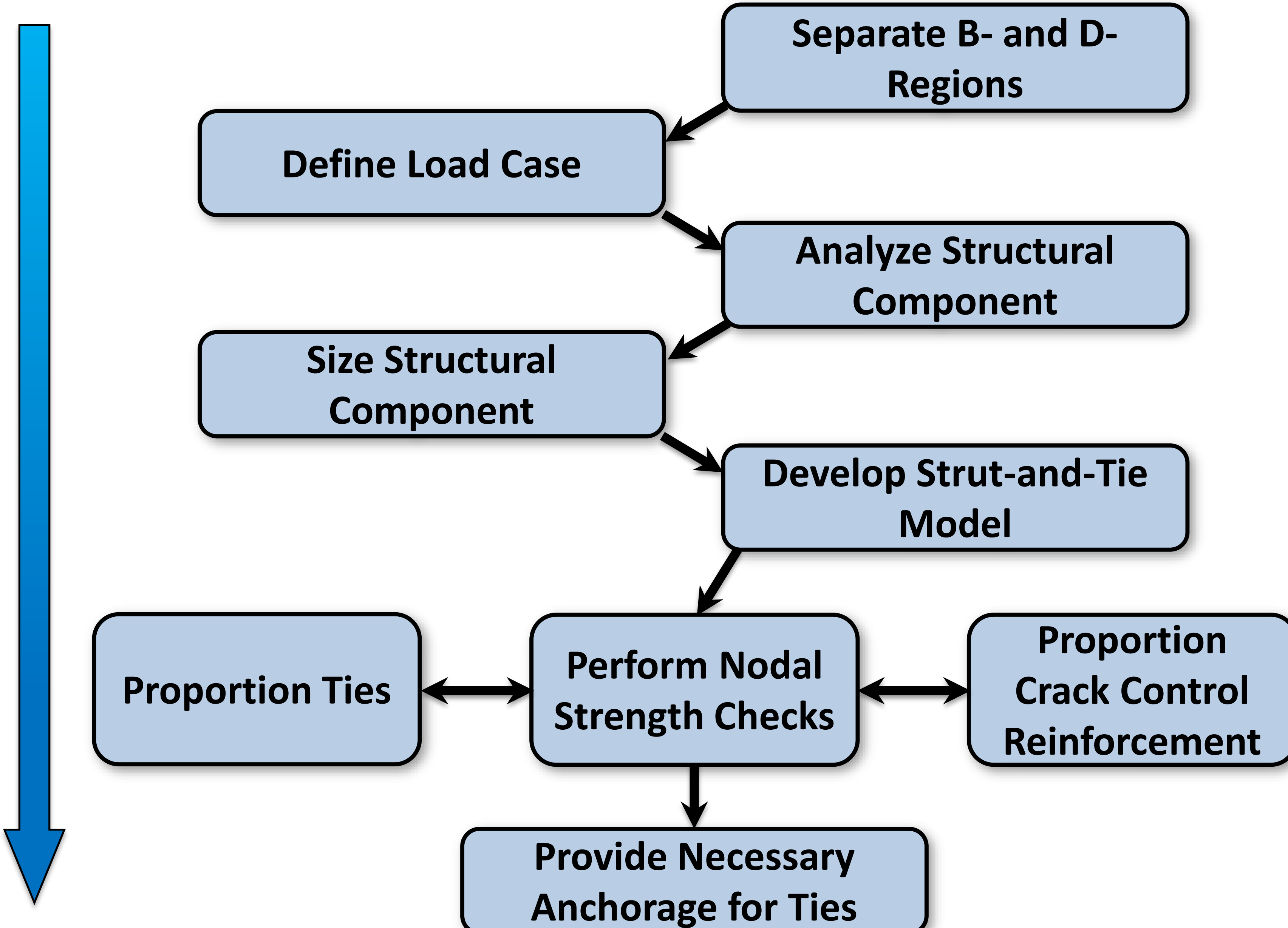
STM FUNDAMENTALS

Bottle-shaped struts

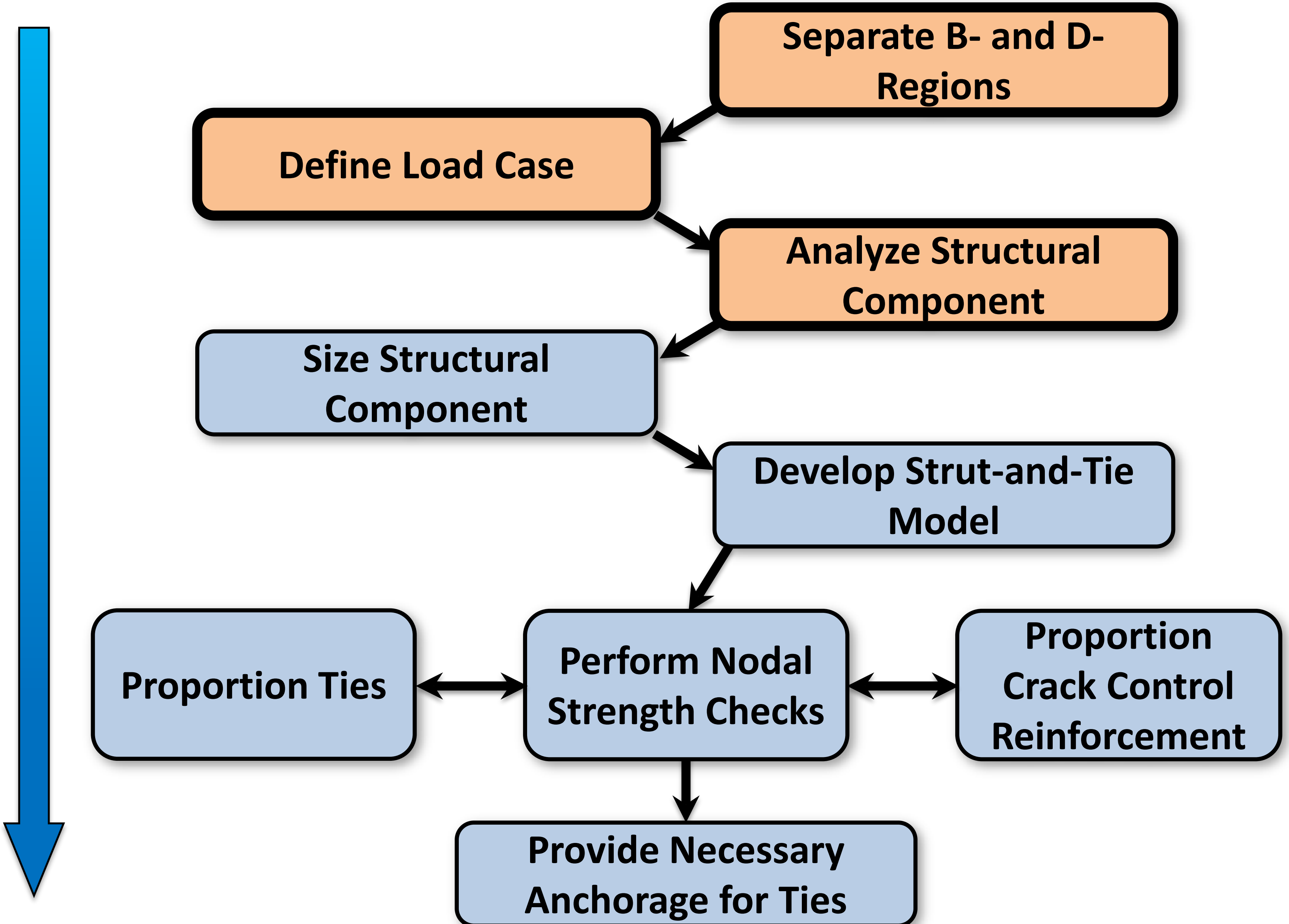
Stresses spread laterally → transverse tension → cracking
Provide reinforcement to control cracking



STRUT-AND-TIE MODEL DESIGN PROCEDURE



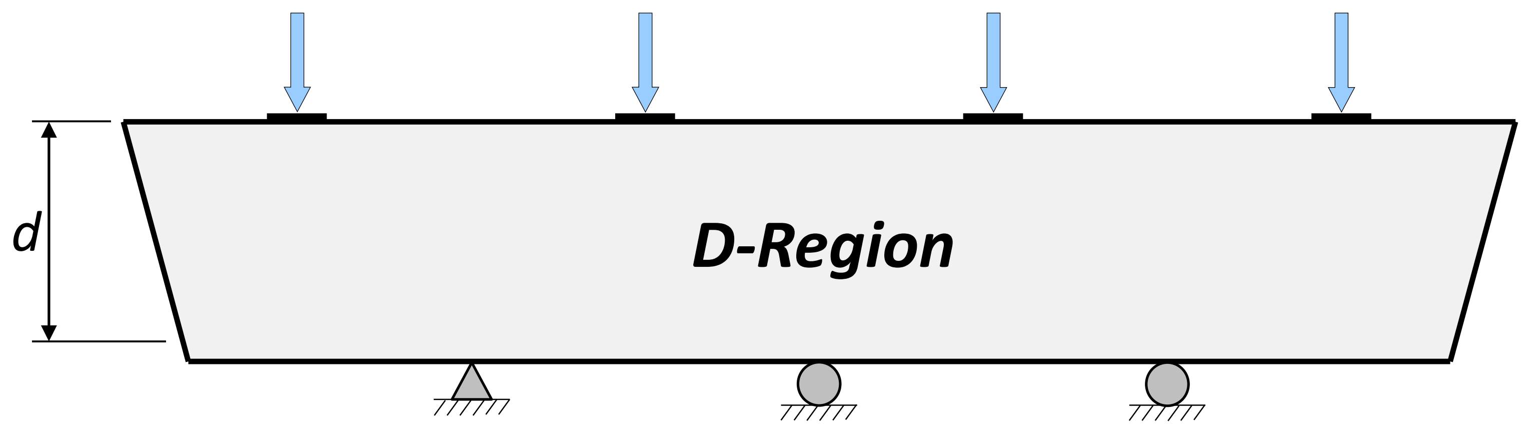
STRUT-AND-TIE MODEL DESIGN PROCEDURE



SEPARATE B- AND D-REGIONS

Apply St. Venant's Principle → *d* away from load or geometric discontinuity

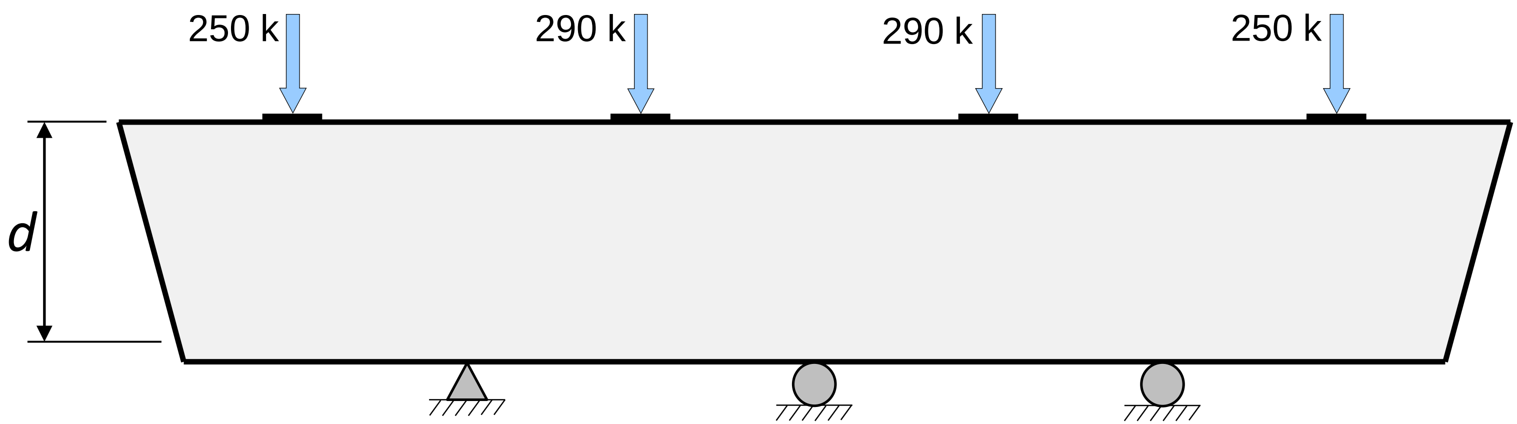
Determine if region is dominated by deep beam behavior or sectional behavior



Entire member is dominated by deep beam behavior

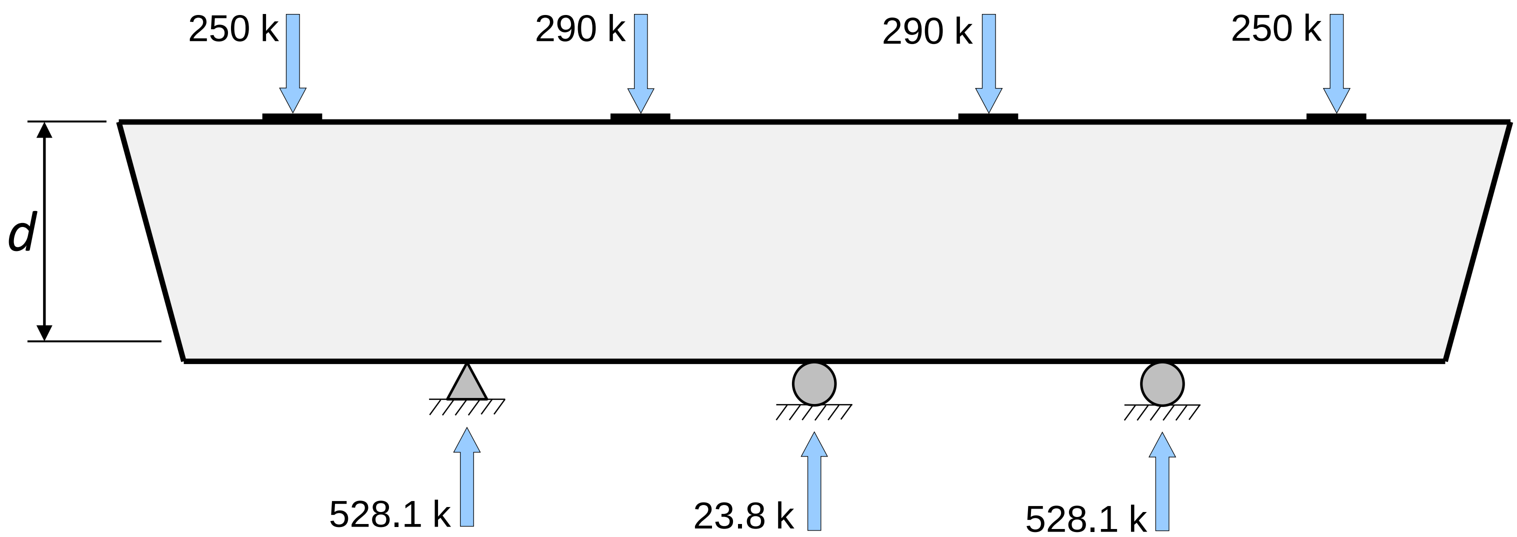
DEFINE LOAD CASE

Apply factored loads to the structural component

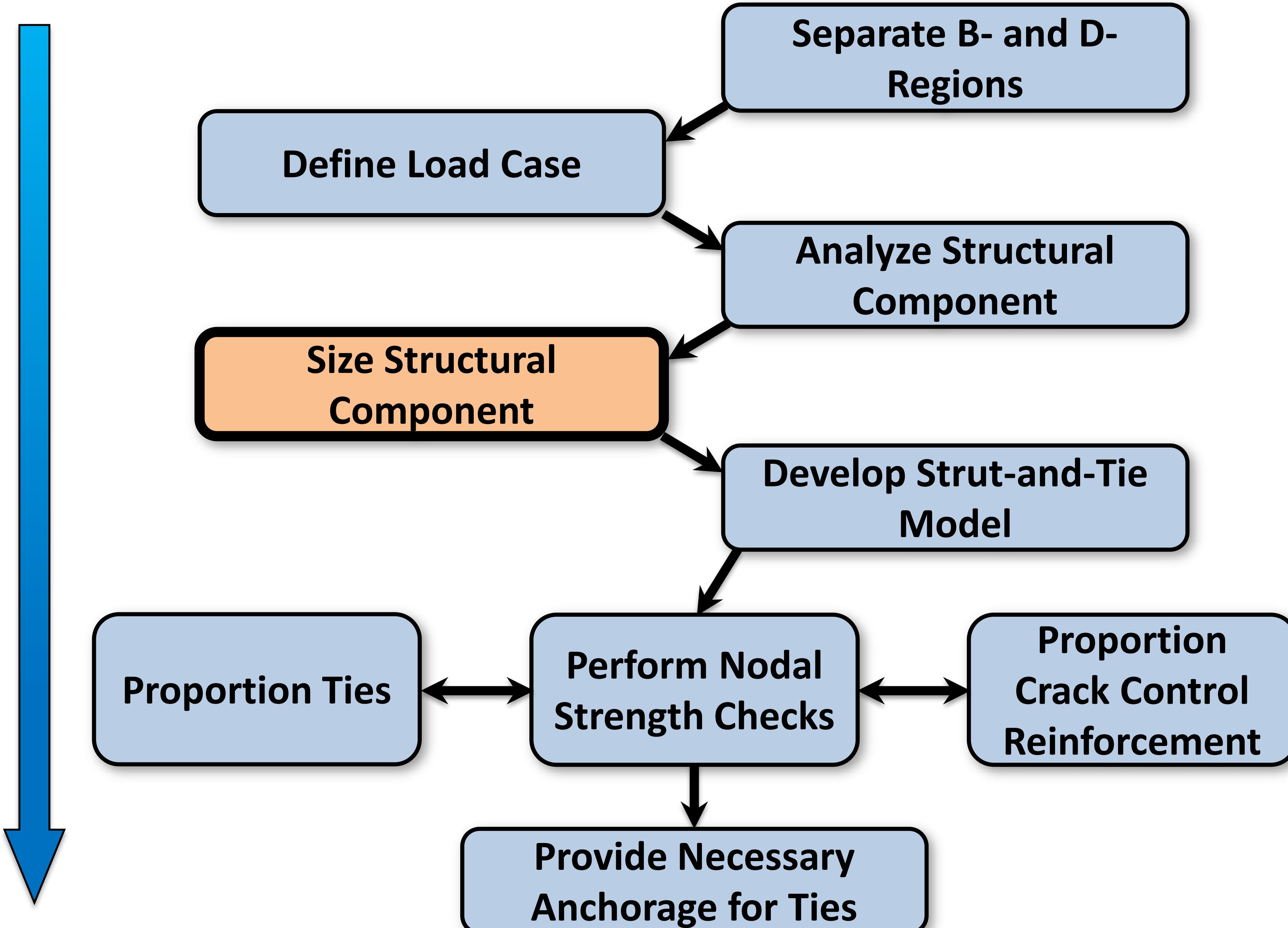


ANALYZE STRUCTURAL COMPONENT

Perform linear-elastic analysis to determine support reactions



STRUT-AND-TIE MODEL DESIGN PROCEDURE



SIZE STRUCTURAL COMPONENT

Choose geometry that reduces the risk of diagonal crack formation under service loads

Determine dimensions so that V_{cr} for the region exceeds the maximum shear force caused by service loads (Birrcher et al., 2009)

$$V_{cr} = \left[6.5 - 3\left(\frac{a}{d}\right)\right]\sqrt{f'_c}\, b_w d$$

but not greater than $5\sqrt{f'_c}\, b_w d$ **nor less than** $2\sqrt{f'_c}\, b_w d$

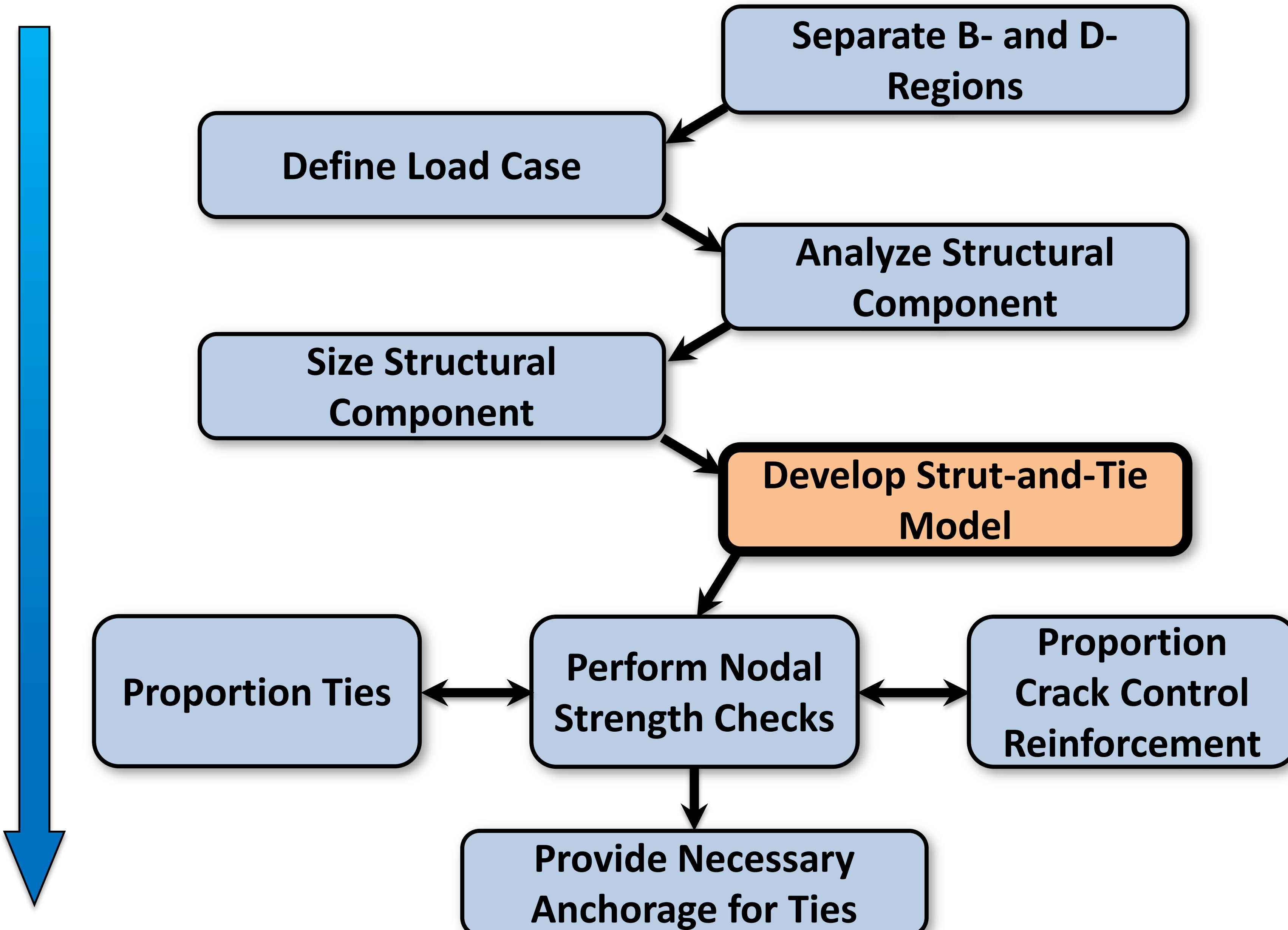
where a = shear span (in.)

d = effective depth of the member (in.)

f'_c = compressive strength of concrete (psi)

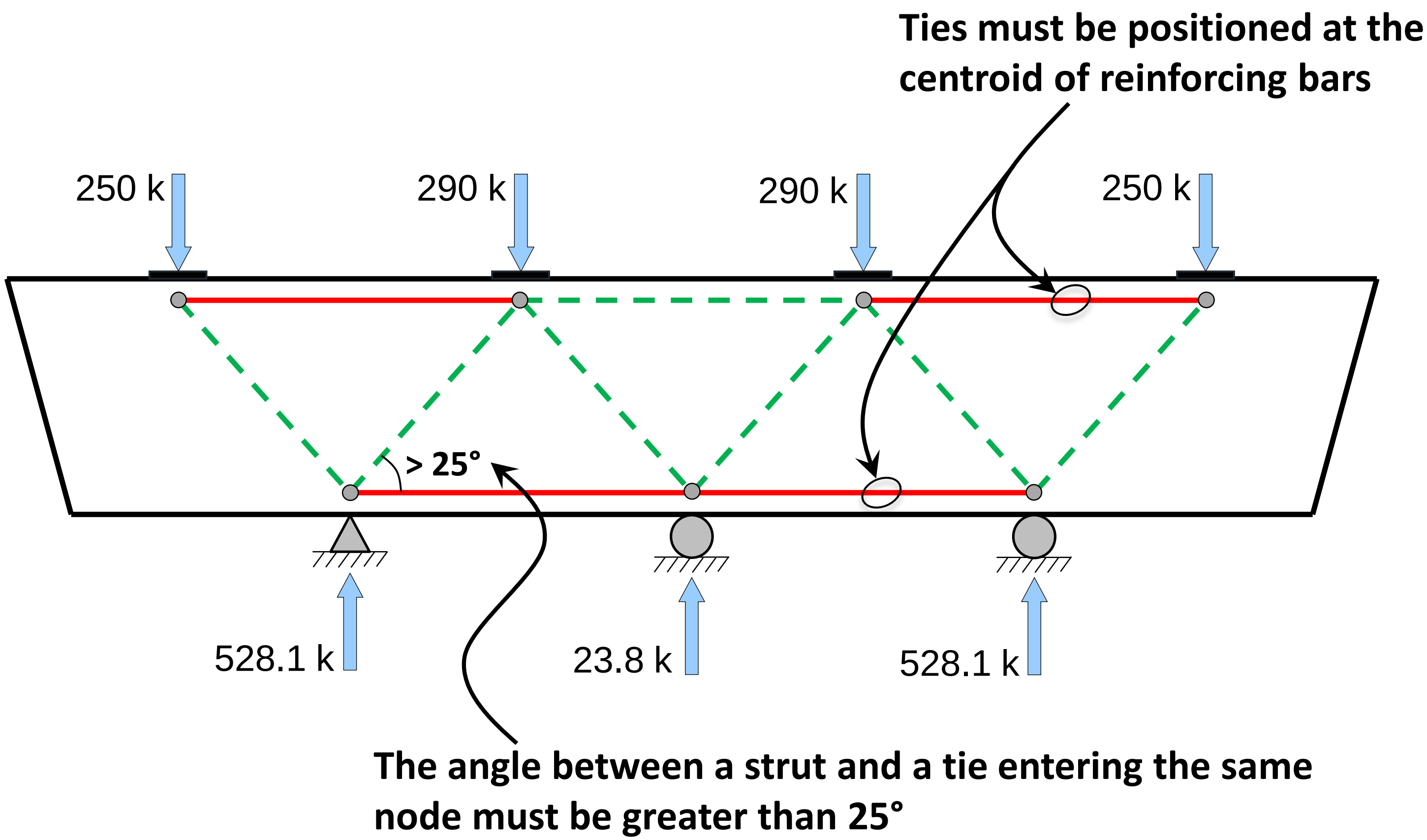
b_w = web width of the member (in.)

STRUT-AND-TIE MODEL DESIGN PROCEDURE



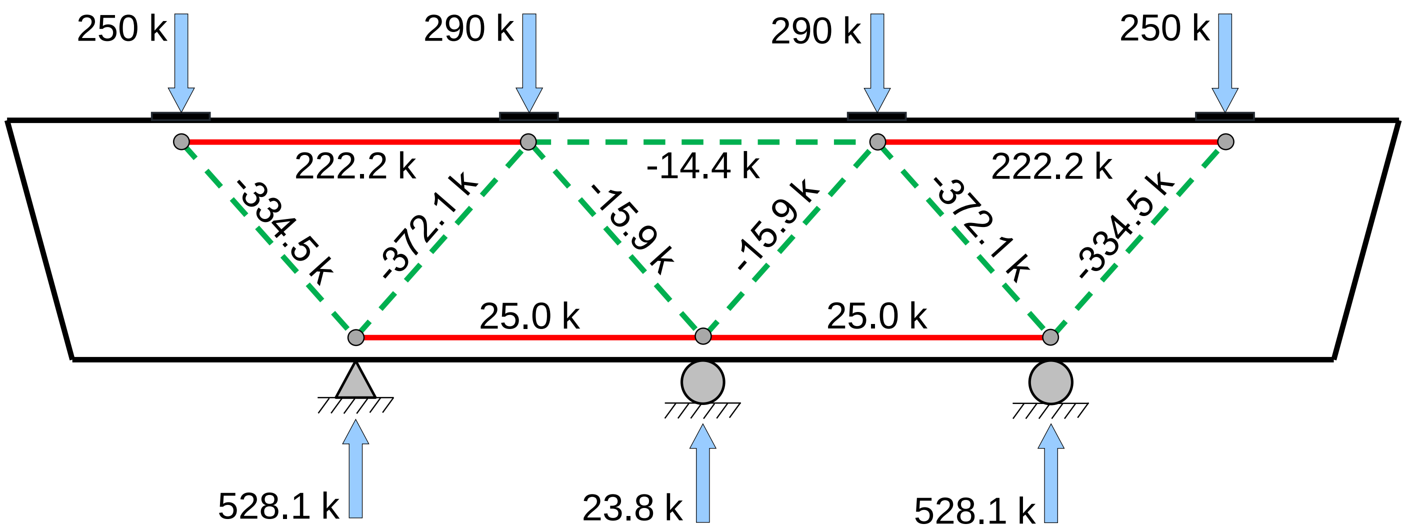
DEVELOP STRUT-AND-TIE MODEL

Place struts and ties to model the flow of forces from the loads to the supports



DEVELOP STRUT-AND-TIE MODEL

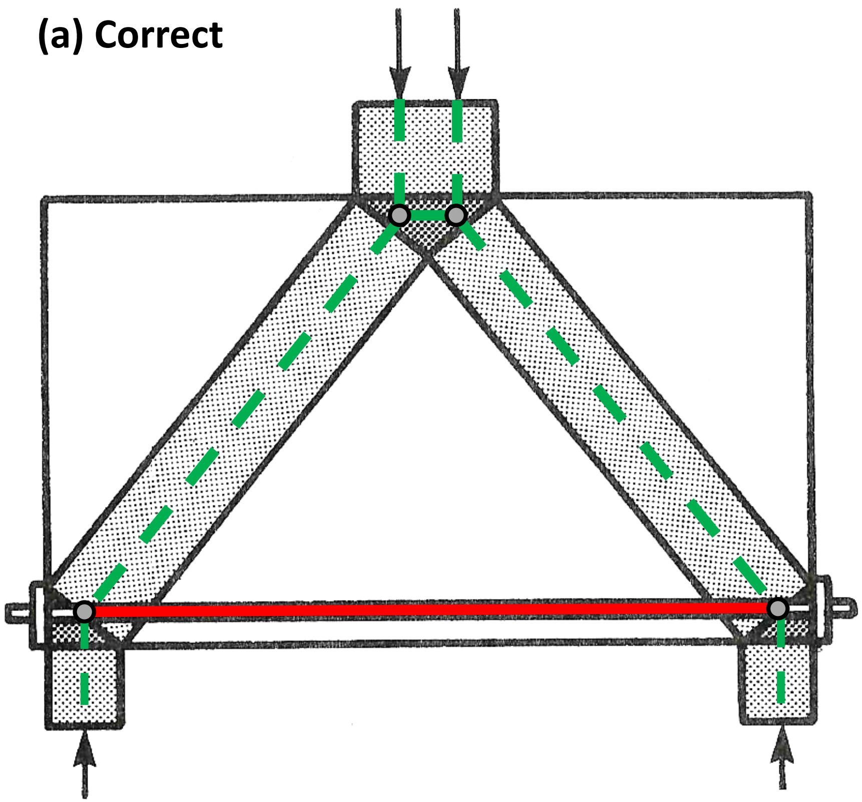
Analyze strut-and-tie model



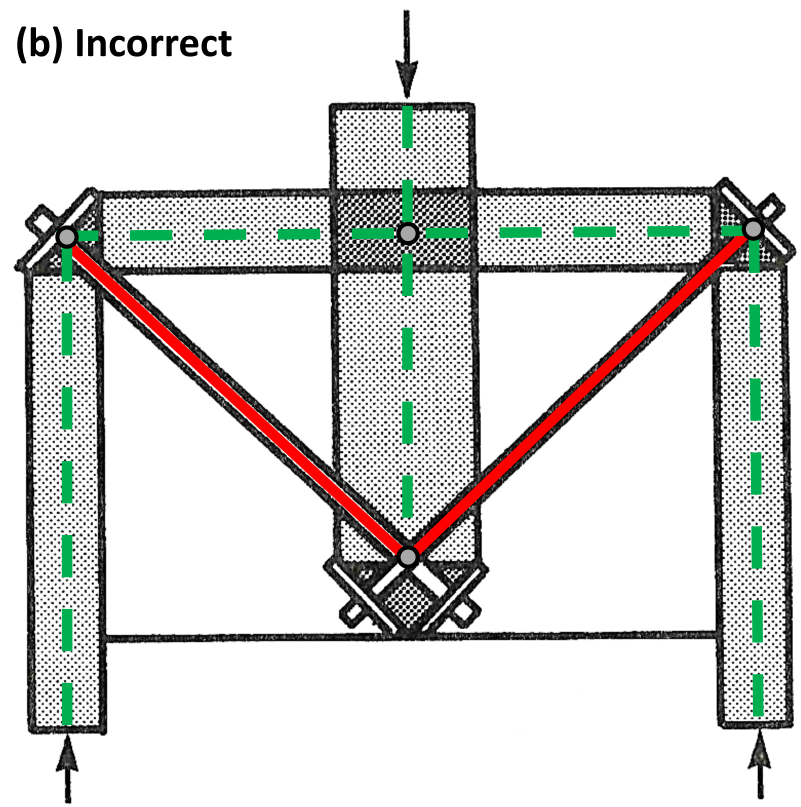
DEVELOP STRUT-AND-TIE MODEL

STM with fewest and shortest ties is the best

(a) Correct

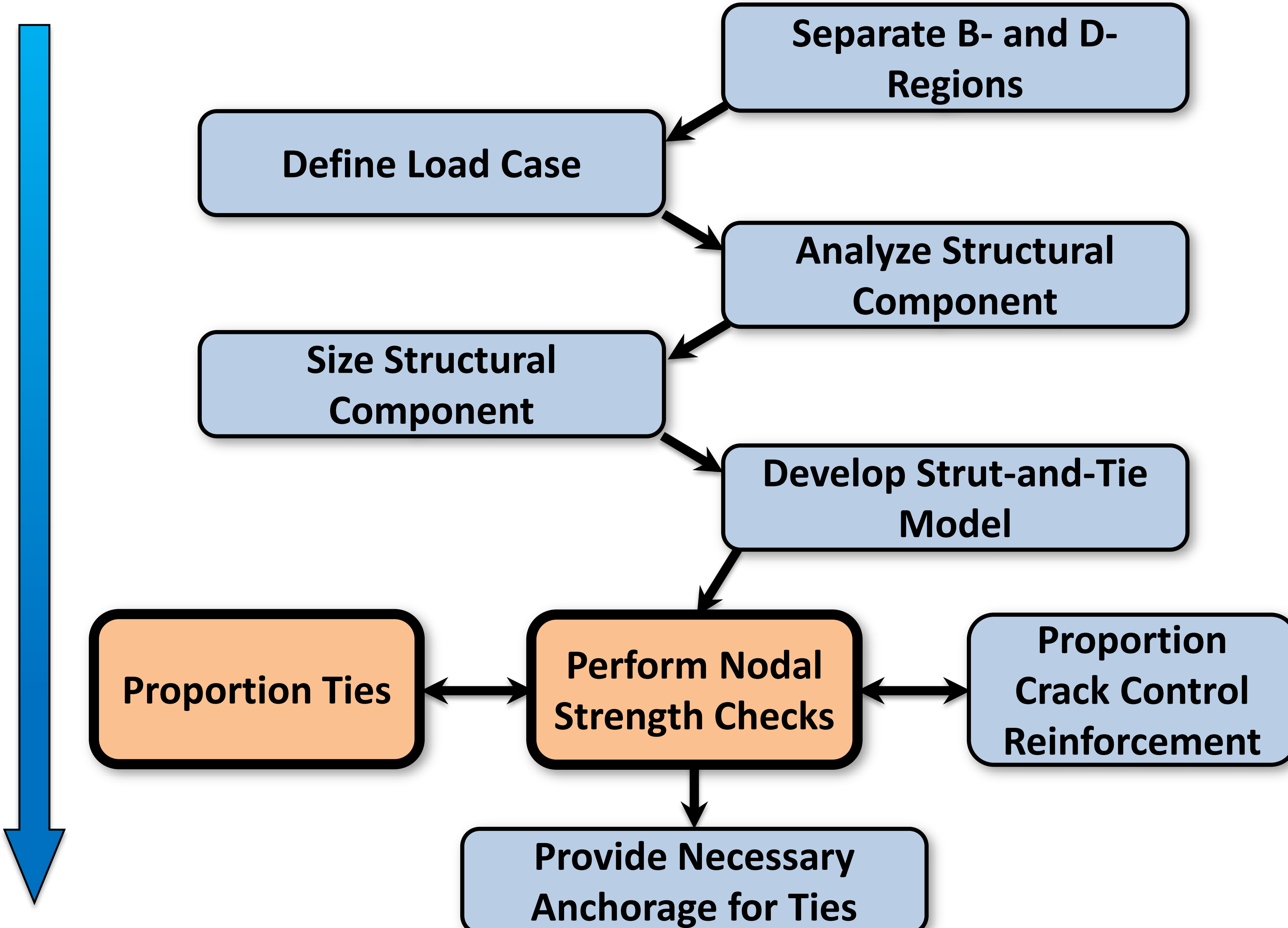


(b) Incorrect



(adapted from MacGregor and Wight, 2005)

STRUT-AND-TIE MODEL DESIGN PROCEDURE



PROPORTION TIES

Determine the area of reinforcement needed to carry the calculated tie forces

$$A_{st} = \frac{P_u}{\phi f_y}$$

where A_{st} = **area of reinforcement needed to carry tie force (in.^{2})**

P_u = **factored force in tie according to the STM (kip)**

f_y = **yield strength of steel (ksi)**

ϕ = **resistance factor (0.90 per AASHTO LRFD)**

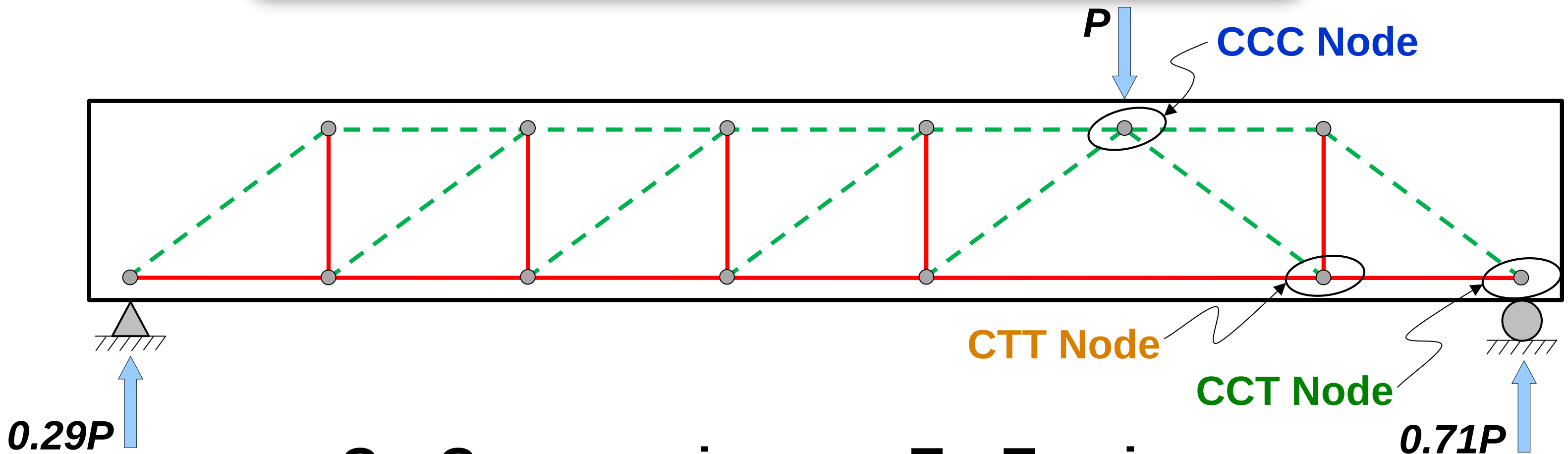
PERFORM NODAL STRENGTH CHECKS

Nodes → Most highly stressed regions (bottleneck of stresses)

Ensure nodal strengths are greater than the forces acting on the nodes to prevent failure

PERFORM NODAL STRENGTH CHECKS

Types of Nodes

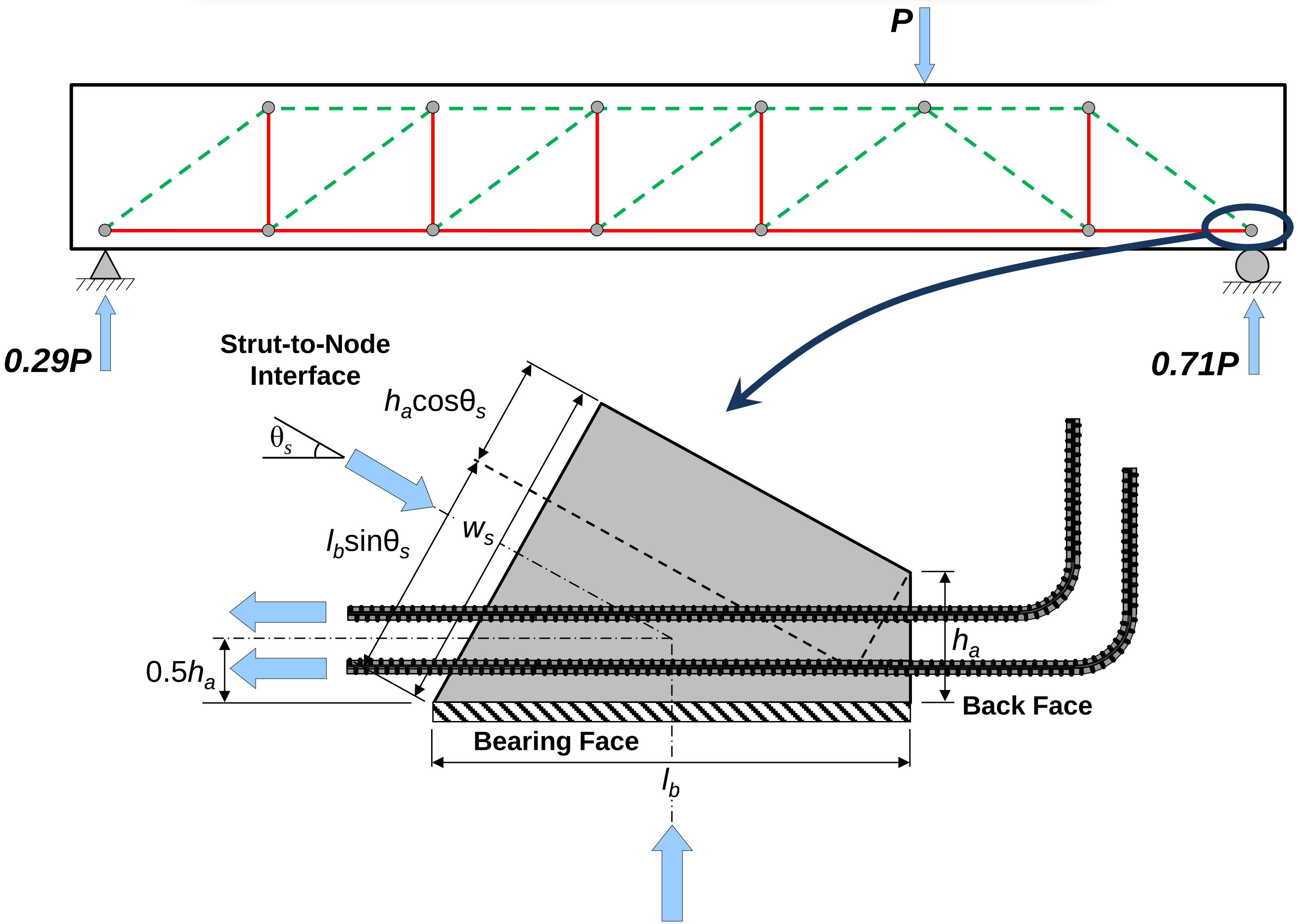


C = Compression **T = Tension**

- **CCC**: **Only struts intersect**
- **CCT**: **Tie(s) intersect node in one direction**
- **CTT**: **Ties intersect node in two different directions**

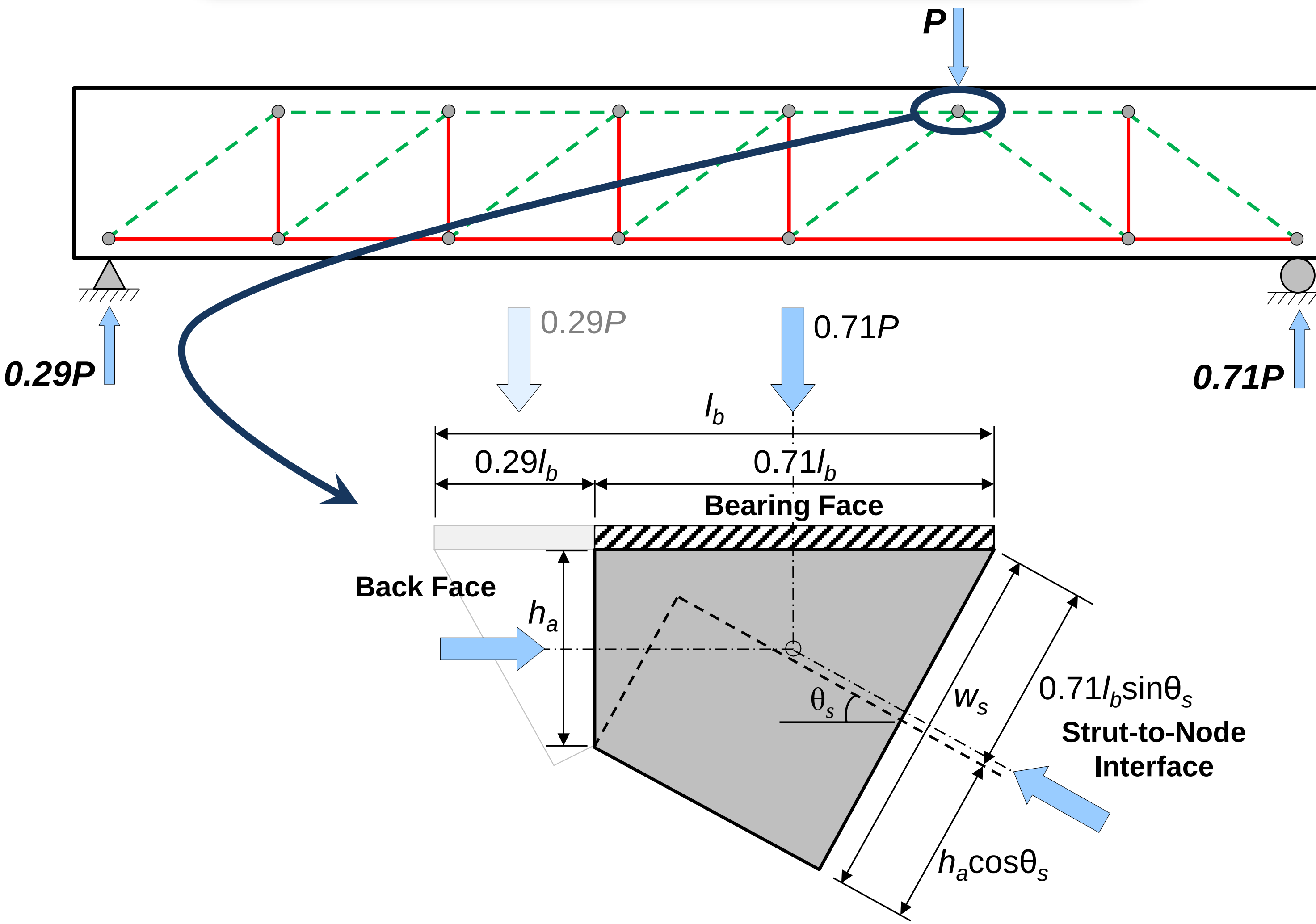
PERFORM NODAL STRENGTH CHECKS

Proportioning CCT Nodes



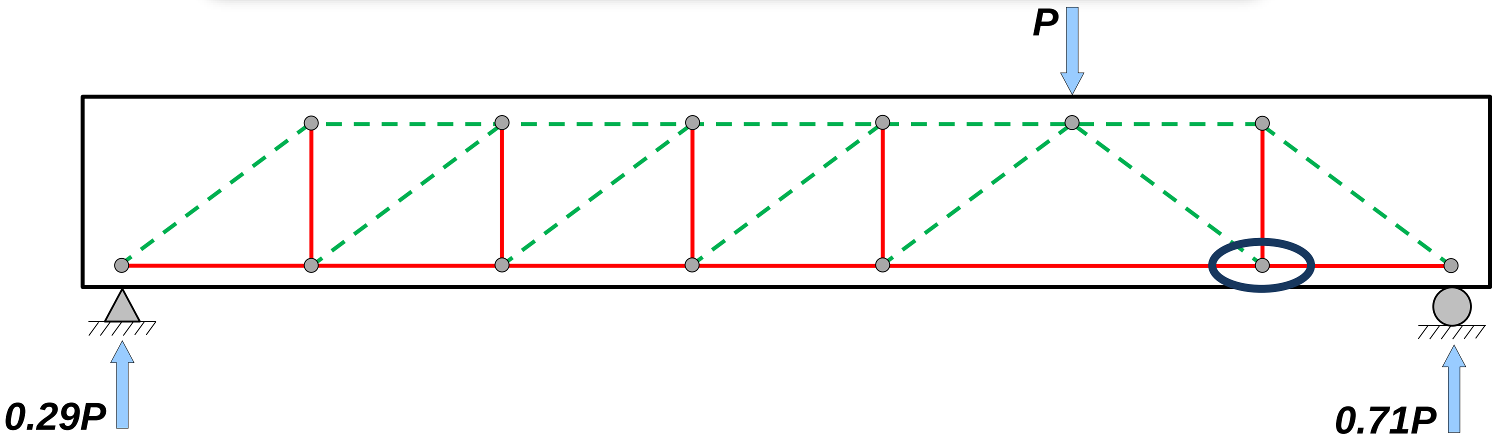
PERFORM NODAL STRENGTH CHECKS

Proportioning CCC Nodes



PERFORM NODAL STRENGTH CHECKS

CTT Nodes



CTT nodes are often *<u>smeared nodes</u>*, or nodes without a geometry clearly defined by a bearing plate or geometric boundaries of the structure

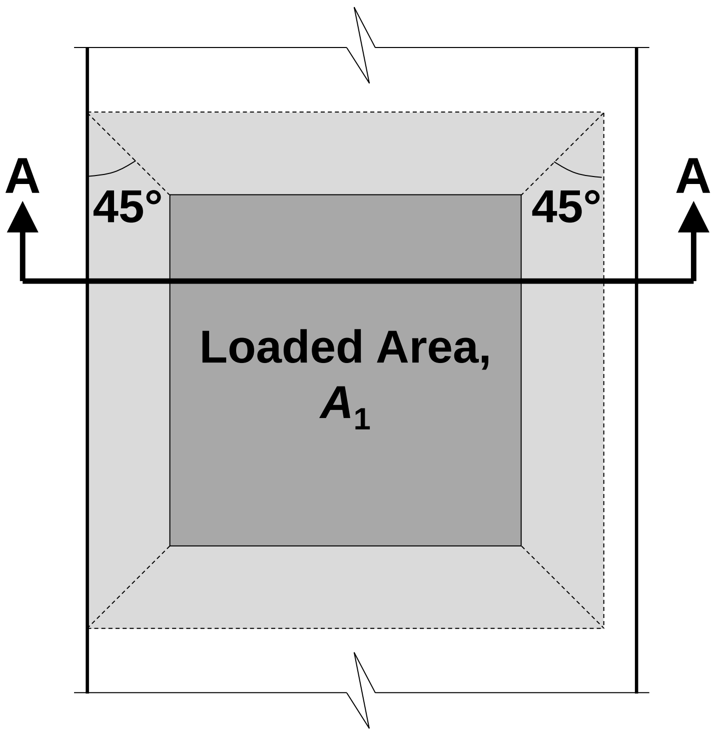
Concrete stresses at smeared nodes do not need to be checked

PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

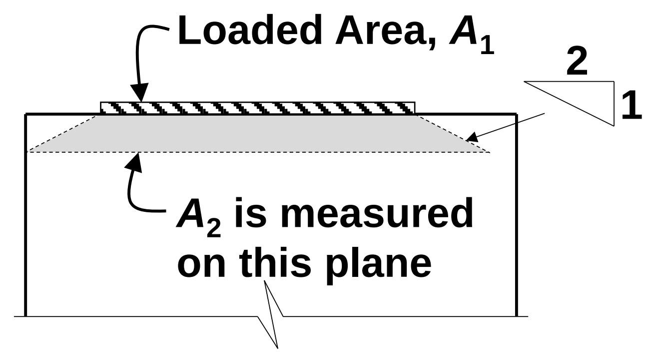
Step 1 – Calculate confinement modification factor, *m*

$$m = \sqrt{A_2/A_1} \leq 2.0$$



Plan View

***m*-factor can be applied to all faces of the node**



Section A-A through Member

PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

Step 2 – Determine concrete efficiency factor, v, for node face under consideration

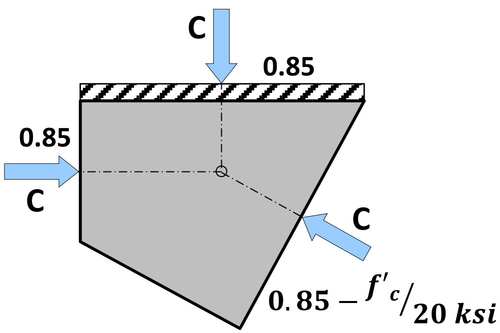
	Node Type		
Face	***CCC***	***CCT***	***CTT***
Bearing Face	$\mathbf{0.85}$	$\mathbf{0.70}$	$\mathbf{0.85 - {f'_c}/{20\ ksi}}$ $\mathbf{0.45 \le \nu \le 0.65}$
Back Face			
Strut-to-Node Interface	$\mathbf{0.85 - {f'_c}/{20\ ksi}}$ $\mathbf{0.45 \le \nu \le 0.65}$	$\mathbf{0.85 - {f'_c}/{20\ ksi}}$ $\mathbf{0.45 \le \nu \le 0.65}$	

If the web crack control reinforcement requirement is not satisfied, use v = 0.45 for the strut-to-node interface

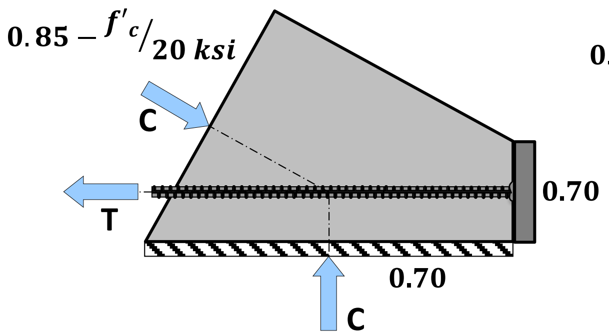
PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

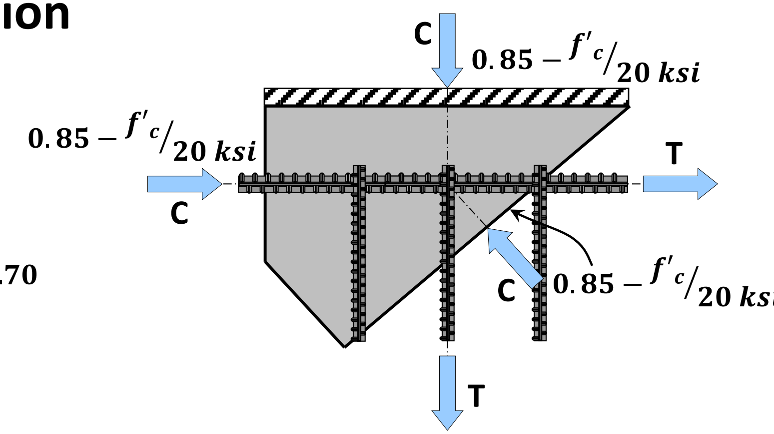
Step 2 – Determine concrete efficiency factor, v, for node face under consideration



CCC Node



CCT Node



CTT Node

More Concrete Efficiency (Stronger)

Less Concrete Efficiency (Weaker)

If the web crack control reinforcement requirement is not satisfied, use v = 0.45 for the strut-to-node interface

PERFORM NODAL STRENGTH CHECKS

Calculating Nodal Strengths

Step 3 – Calculate the design strength of the node face, ϕ*P$_n$**

$$\phi \cdot P_n = \phi \cdot f_{cu} \cdot A_{cn}$$

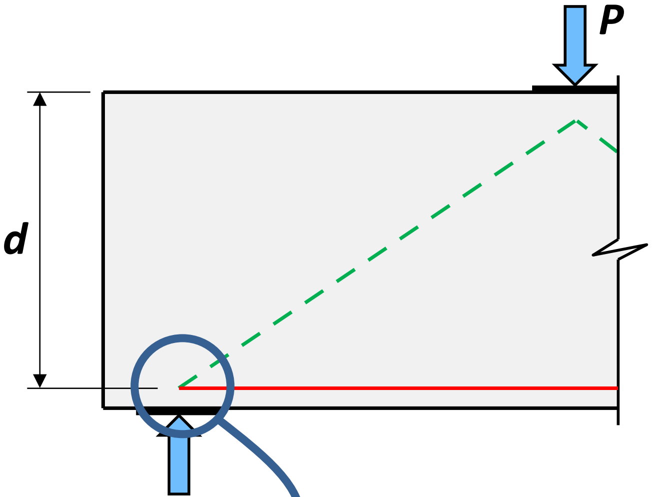
$$f_{cu} = m \cdot \nu \cdot f'_c$$

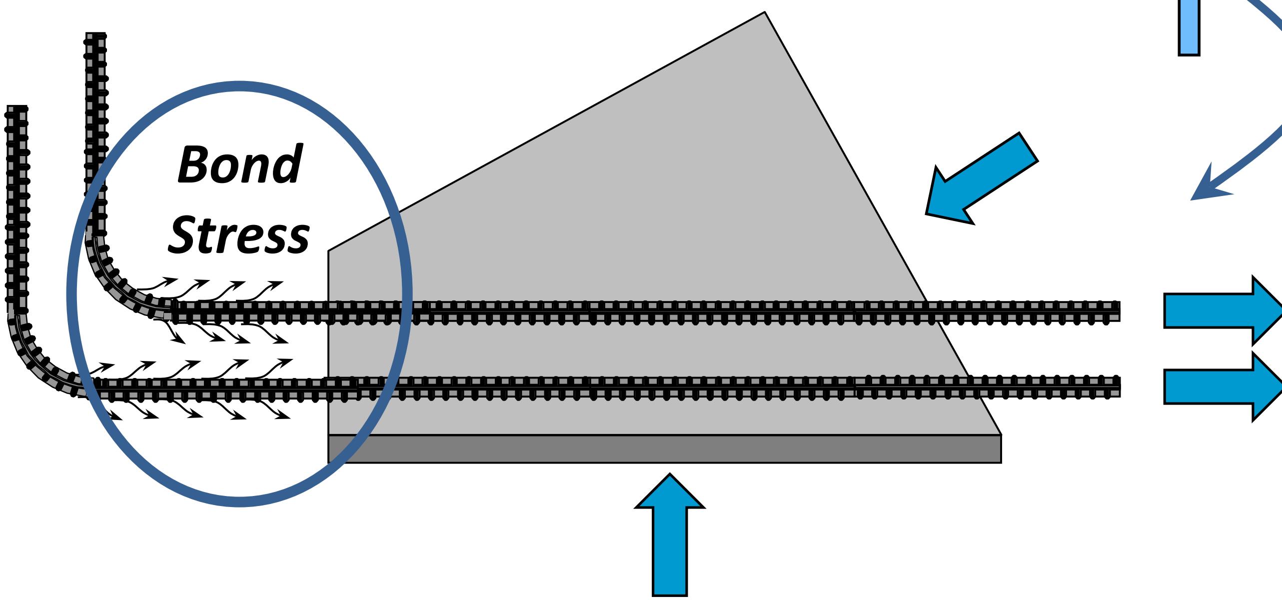
where f_{cu} = limiting compressive stress (ksi)
ϕ = resistance factor for compression in STMs (0.70 per AASHTO LRFD)
A_{cn} = effective cross-sectional area of the node face (in.^{2})

Ensure the design strength, ϕP_n, is greater than or equal to the factored force, P_u, acting on the node face:

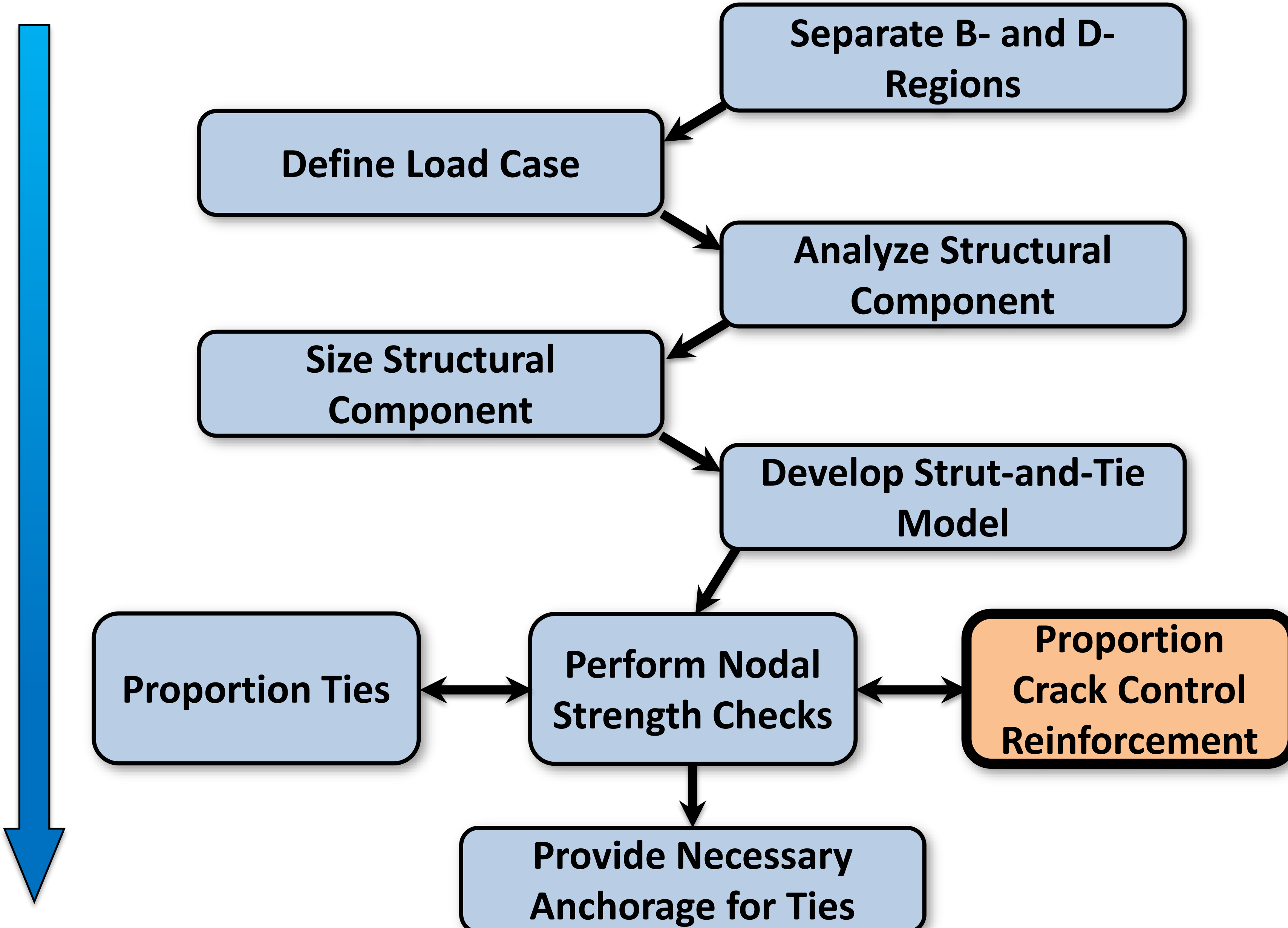
$$\phi P_n \geq P_u$$

PERFORM NODAL STRENGTH CHECKS





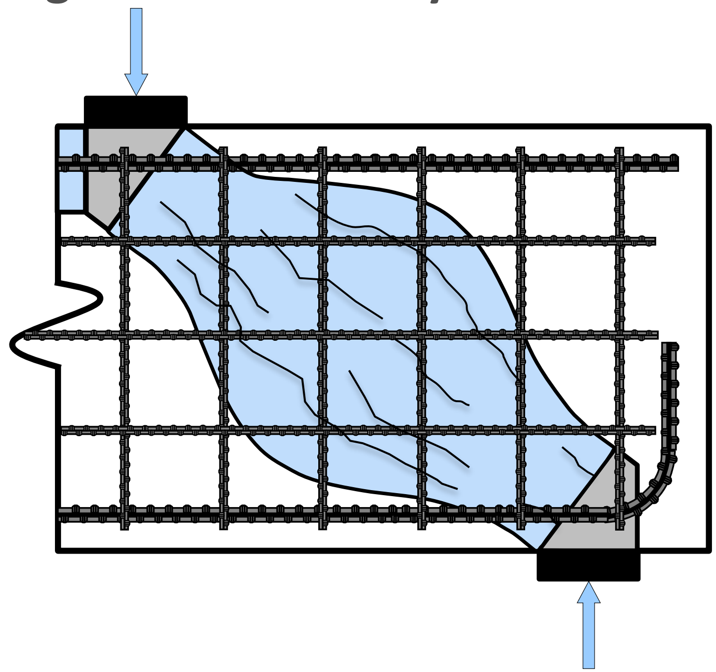
STRUT-AND-TIE MODEL DESIGN PROCEDURE



PROPORTION CRACK CONTROL REINFORCEMENT

Provide distributed orthogonal reinforcement that can:

- **Carry tensile stress transverse to bottle-shaped struts**
- **Restrain bursting cracks caused by this tensile stress**



- **Increase ductility by allowing redistribution of stresses**

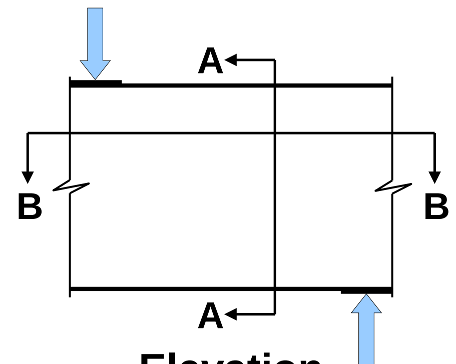
PROPORTION CRACK CONTROL REINFORCEMENT

Provide 0.3% reinforcement in each orthogonal direction
(with the exception of slabs and footings)

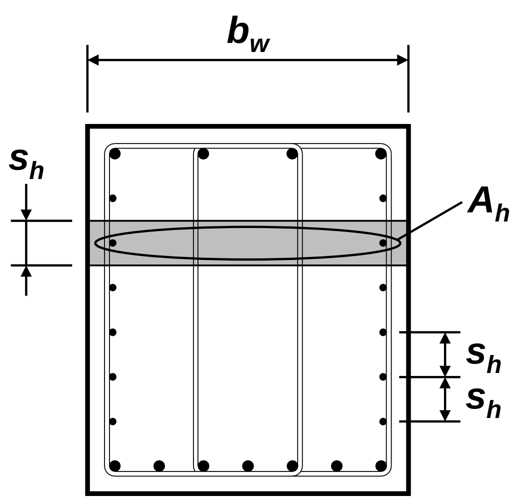
- Evenly space reinforcement as shown

$$\frac{A_v}{b_w s_v} \geq 0.003 \qquad \frac{A_h}{b_w s_h} \geq 0.003$$

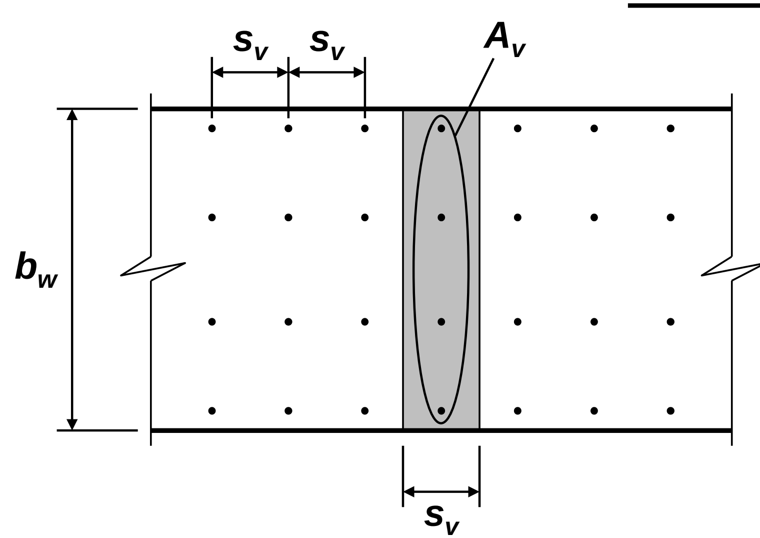
s_v and s_h shall not exceed $d/4$ or 12 in.



Elevation

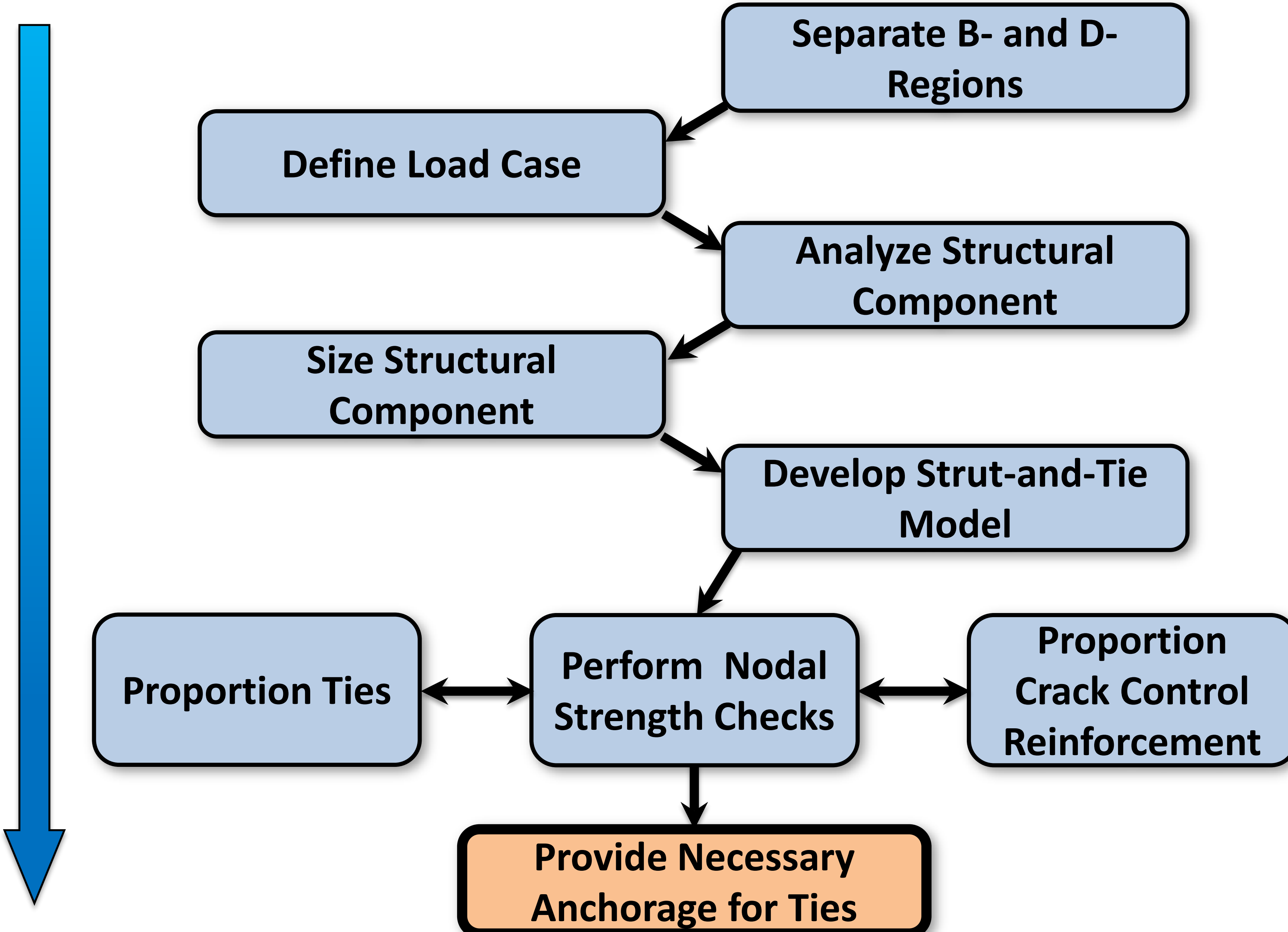


Section A-A



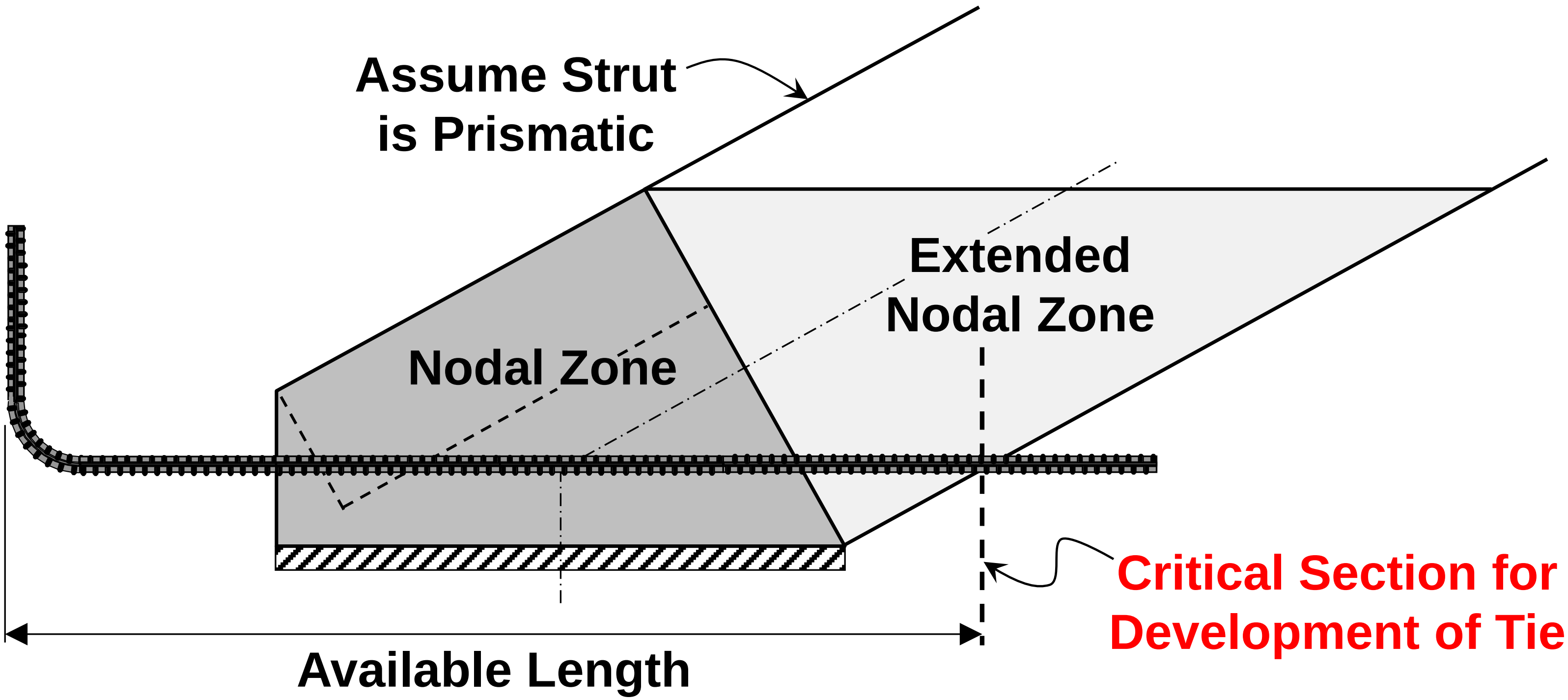
Section B-B

STRUT-AND-TIE MODEL DESIGN PROCEDURE

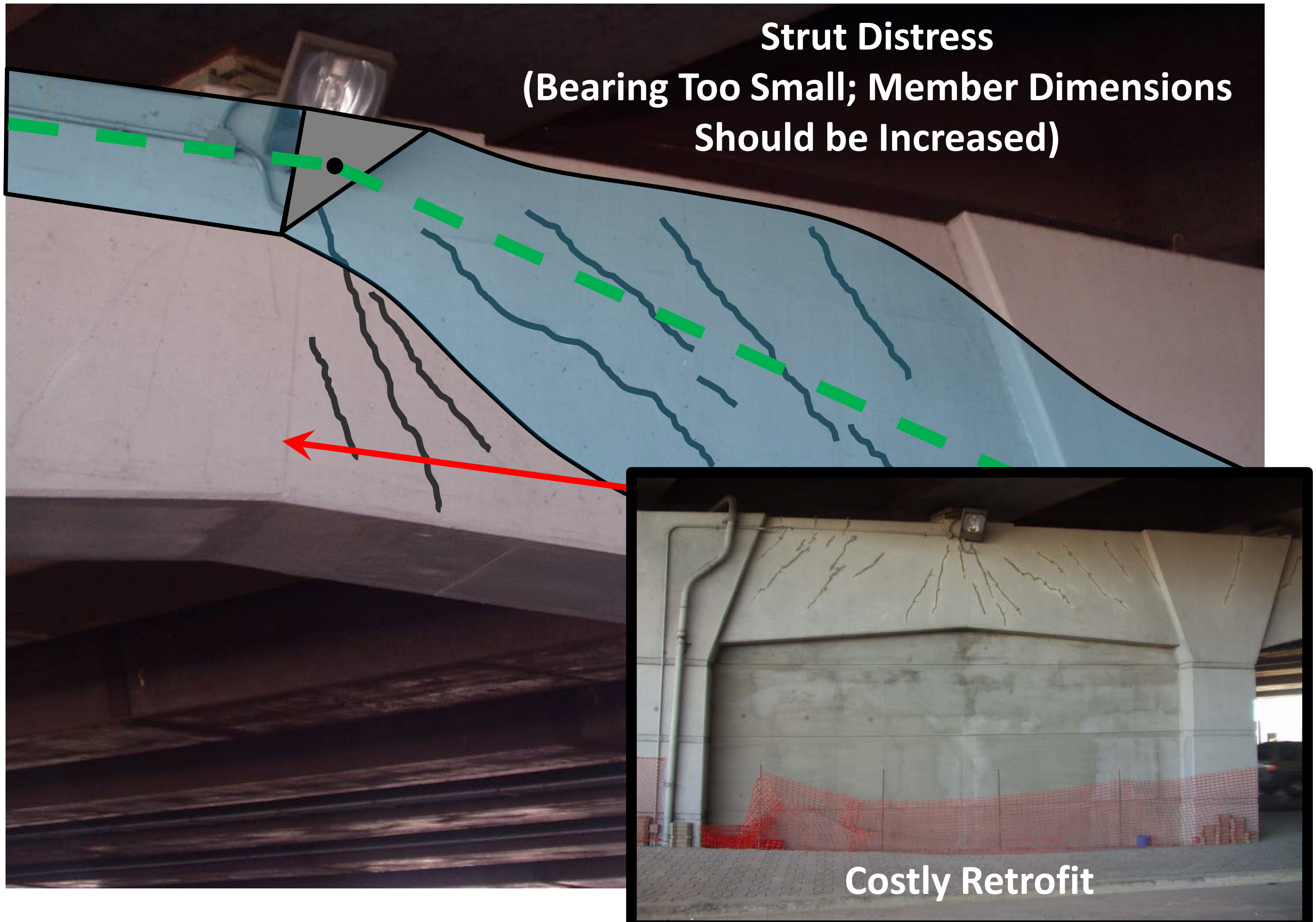


PROVIDE NECESSARY ANCHORAGE FOR TIES

Reinforcement must be fully developed at the point where the centroid of the bars exits the *extended nodal zone*



FIELD ISSUES AND THE IMPACT OF STM

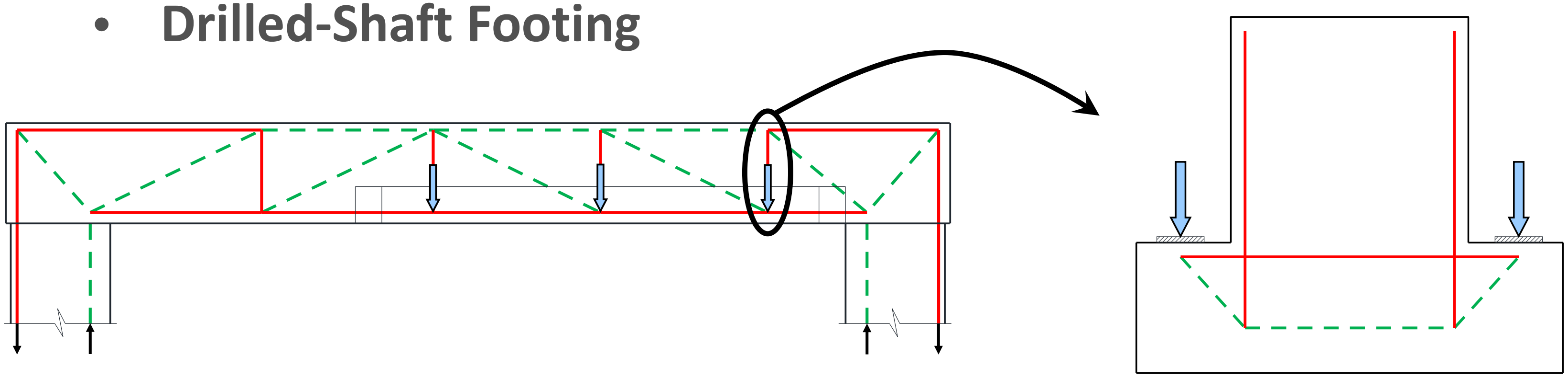


STM GUIDEBOOK WITH DESIGN EXAMPLES

http://www.utexas.edu/research/ctr/pdf_reports/5_5253_01_1.pdf

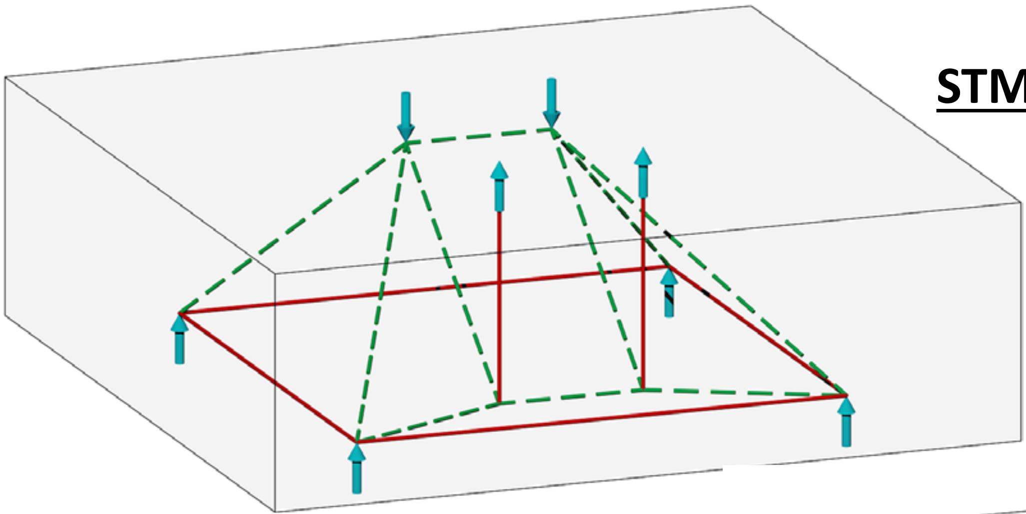
- **Step-by-step introduction to strut-and-tie modeling design procedure in accordance with AASHTO LRFD**

- **5 STM design examples of bridge components**
  - **Five-Column Bent Cap of a Skewed Bridge**
  - **Cantilever Bent Cap**
  - **Inverted-T Straddle Bent Cap (Moment Frame)**
  - **Inverted-T Straddle Bent Cap (Simply Supported)**
  - **Drilled-Shaft Footing**

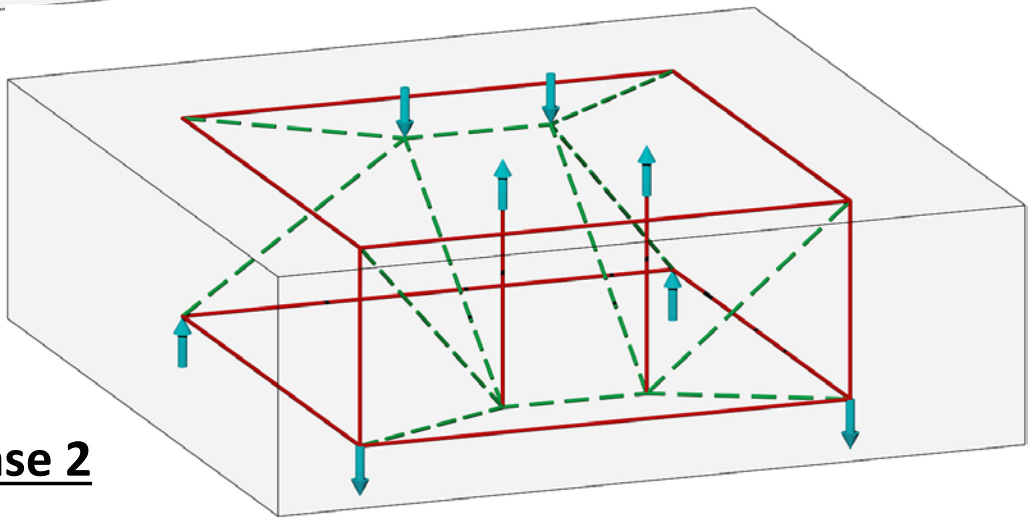


STM GUIDEBOOK WITH DESIGN EXAMPLES

- **3D STM - Drilled-shaft footing design example**



<u>STM for Load Case 1</u>



<u>STM for Load Case 2</u>

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THANK YOU!